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Logical Investigations in the Lvov-Warsaw School and Their Worldwide Influence[1]

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Abstract: The aim of the article is to concisely discuss the creative contribution of the logicians of the Lvov-Warsaw School, which led to developments in Polish and world logic in the 20th century and which continue to influence its functioning on the international stage.

Key words: Lvov-Warsaw School, Warsaw School of Logic, Polish logic, logical heritage and influence of the Lvov-Warsaw School

1. Introduction[1]

Contemporary world literature dealing with logic and its history very often features the term "Polish logic." This term was first applied by Storrs McCall to underline the great contribution of Polish logicians of the interwar period, in particular members of the Warsaw School of Logic (hereafter WSL), to the development of logic internationally.[2] The WSL was a renowned branch of the Lvov-Warsaw School (hereafter LWS) – the most important movement in the history of Polish philosophy. This philosophical school, established by Kazimierz Twardowski, laid foundations for formal logic developed later in the WSL and in certain domains in which Polish scientists played a pioneering role, outstanding on a global scale.

[1] The author wishes to thank the reviewers of the paper for comments and suggestions. Several remarks that they made led to some additions or improvements in the text.

[2] S. McCall, ed., *Polish Logic in 1920–1939: Papers by Ajdukiewicz, Chwistek, Jaśkowski, Jordan, Leśniewski, Słupecki, Sobociński, and Wajsberg*, introd. T. Kotarbiński, Clarendon Press, Oxford 1967.

Much has already been written on the main pillars, ideas and significance of the WSL.[3] Thus, in the present work I do not limit myself to merely reviewing the achievements of representatives of the WSL, my intention being to present the general output of the logical thought that was formed in the LWS as well as its legacy, which we and the School's followers were left by the logicians of the LWS to make use of and popularize.

The successive parts of the article contain the following: a short description of the formation of logic in the LWS (section 2), presentation of the most significant achievements of mathematical logicians (section 3), review of selected achievements of logician-philosophers of the LWS (section 4), discussion of the influence of the output of Warsaw-based logicians of the interwar period on the development of logic in the 20th century (section 5), and cultivation of the tradition of Polish logic and its fertile impact on contemporary scientific research in the world (section 6).

2. Formation of the Scientific Logical Thought in the LWS

Logic had not enjoyed much interest in Poland before the 20th century. The emergence of interest in logic and development of the discipline are traditionally connected with the beginning of a new era in philosophy in Poland and the person of Kazimierz Twardowski, as well as with his appointment as Chair of Philosophy at Lvov University in 1895. Thanks to this great thinker, the first modern school of philosophy was established, known as the Lvov School and later – the Lvov-Warsaw School. Its activity promoted the development of logic, especially mathematical logic. It this domain, Polish scientists played a pioneering and momentous role

[3] See, e.g., J. Woleński, *Filozoficzna Szkoła Lwowsko-Warszawska* [Philosophical Lvov-Warsaw School], Państwowe Wydawnictwo Naukowe, Warszawa 1985; J. Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, Kluwer, Dordrecht 1989; J. Woleński, *Lvov-Warsaw School*, in: *The Stanford Encyclopedia of Philosophy*, ed. E.N. Zalta, Winter 2015 ed., URL: https://plato.stanford.edu/archives/win2015/entries/lvov-warsaw/; U. Wybraniec-Skardowska, *Polish Logic: A Few Lines from a Personal Perspective*, ILLC Prepublication Series PP2009-31, ILLC, University of Amsterdam, Amsterdam 2009; U. Wybraniec-Skardowska, *Introduction: The School. Its Genesis, Development and Significance*, in: *The Lvov-Warsaw School: Past and Present*, eds. A. Garrido, U. Wybraniec-Skardowska, Springer, Birkhäuser, Cham 2018, pp. 3–14; U. Wybraniec-Skardowska, *The Warsaw School of Logic: Main Pillars, Ideas, Significance*, "Studia Humana" 2024, Vol. 13, No. 1, pp. 17–27; M. Tkaczyk, U. Wybraniec-Skardowska, *Polska logika* [Polish Logic], in: *Encyklopedia filozofii polskiej* [Encyclopedia of Polish Philosophy], Vol. 1, Polskie Towarzystwo Tomasza z Akwinu, Lublin 2011, pp. 880–890.

on a global scale. Undoubtedly, the shaping and development of logical creativity would not have been possible without the route delineated by Twardowski's mature approach, without his personal involvement and guarantee of the highest standards of research work, or forming the attitude of creative inquiry in the members of the LWS – one that promoted an image of the human being as one who takes pride in intellectual activity and in a distinctive approach to understanding scientific progress.

Twardowski's School was characterized by accentuating an exact and clear way of thinking and formulating thoughts, their exchange, proper and correct justification of theorems as well as application of modern logical-methodological means. Consequently, works by representatives of the School were characterized by a high level of scientific culture, as regards both their philosophical and logical content and presentation.

In the programme of philosophy of the School, logic in a broad sense, that is, formal logic, semantics and methodology of sciences, constituted a significant element. Although Twardowski himself was not a logician, he lectured in logic, in particular algebra of logic, and inspired some of his students to take up this direction in philosophy. Therefore, already in the first Lvov period of the LWS, research in the field of logic was conducted by at least six of Twardowski's students: Jan Łukasiewicz (principles of contradiction and the excluded middle, logical values, logical entailment, an important monograph on the principle of contradiction); Kazimierz Ajdukiewicz (logical entailment); Tadeusz Czeżowski (theory of antinomies); Tadeusz Kotarbiński (logical values, principles of non-contradiction and the excluded middle); Stanisław Leśniewski (theory of antinomies, principles of contradiction and the excluded middle, logical values); and Zygmunt Zawirski (theories of modality). All of them exerted a considerable influence on the development of Polish logic. However, it was Łukasiewicz and Leśniewski, as well as their students, who made the most significant contributions to the international success of Polish logic, particularly mathematical logic.

Łukasiewicz and Leśniewski are regarded as the founders of a peculiar branch of the LWS – the Warsaw School of Logic, which was active in the years 1918–1939. The beginnings of the WSL are connected with the reactivation of Warsaw University (1916) and regaining independence by Poland (1918). Łukasiewicz and Leśniewski – by education philosophers – became professors of the University of Warsaw at the Faculty of Mathematical and Natural Sciences. Both met with openness on the part of the mathematicians based there, who, while implementing the programme introduced by Zygmunt Janiszewski in 1918, empha-

sized the role of the problem area of foundations of mathematics, particularly taking mathematical logic and set theory into account. Thus, Łukasiewicz and Leśniewski, during the first period of their activity in the WSL, concentrated on teaching mathematical logic mainly to mathematicians, but also to philosophers. The manner of practising logic by the founders of the WSL and their students was shaped by Twardowski's philosophical programme, while also drawing on Janiszewski's mathematical programme. In this way, logic in the LWS developed along two foundational lines: philosophical and mathematical.

It is worth mentioning that mathematical logic as practised in the WSL was considered to be an autonomous discipline, independent of both philosophy and mathematics. This autonomy did not hinder close cooperation between the Warsaw-based logicians and their mathematical and philosophical colleagues; rather, it stimulated their scientific creativity and offered motivation for their various theoretical investigations into formal systems. The assumption of logic's autonomy significantly strengthened Polish logic, leading to the sophisticated development of counting techniques (similar to those used by mathematicians), and – through those applications – it enabled the representatives of the School to achieve important findings relating to formal systems. At the same time, the logical research was often reinforced by philosophical motivation and the results obtained were philosophically seminal. There also dominated the conviction that logic performs a vital societal function by opposing all attempts at irrationalism.

The WSL had its own special research programme and a framework of practising logic in a very precise, intelligible and intuitive way.[4] The implementation of these principles yielded outstanding scientific results and a substantial rise in the significance of Polish logic as well as the attractiveness of logical studies.

3. Key Constituents of the Logical Output of Representatives of the WSL

The WSL has passed down to us the unique heritage of the logicians grouped in the LWS. Below, we will briefly discuss the most significant results of the creative logical thought of representatives of the WSL.

[4] J. Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, op. cit.; A. Brożek, *Jan Łukasiewicz's Program of the Logicization of Philosophy: Its Genesis, Content and Realization*, "Synthese" 2022, Vol. 200, No. 3, pp. 1–24.

The group of students of Łukasiewicz and Leśniewski – the founders of the WSL – included: Alfred Tarski (considered one of the most outstanding logicians of the 20th century), Stanisław Jaśkowski, Czesław Lejewski, Adolf Lindenbaum, Andrzej Mostowski, Mojżesz Presburger, Jerzy Słupecki, Bolesław Sobociński, and Mordechaj Wajsberg. Together, they formed the nucleus of the WSL in the years 1918–1939. The most significant achievements of the School, constituting the valuable heritage of Polish logic, are listed below:[5]

- **Elaboration of the elementary methods of mathematical logic:** defining a set of well-formed expressions of the language of logic, axiomatization, formalization of logical systems.
- **Construction and research on propositional logical systems; Łukasiewicz's first many-valued systems; Polish notation:** Many logical systems were created and formally examined. A number of tools or techniques to study the properties of these systems were created and developed. Łukasiewicz provided a new axiomatic system for the classical propositional calculus as well as a proof of its completeness. Inspired by Aristotle's formulation of the problem of the logical value of sentences describing future chance events, he discovered the first many-valued logical systems. Łukasiewicz proposed and applied, at the same time, parenthesis-free symbolism – very convenient in theoretical considerations – later called Polish notation.
- **Creation and development of tools to examine the properties of logical systems:** The matrix method of constructing propositional calculi was created within the WSL; the concept of a logical matrix also materialized here, while the theory of logical matrices was being developed (Łukasiewicz, Tarski, Lindenbaum); Lindenbaum proved the theorem of the existence of matrices adequate to any propositional logic closed with respect to substitution; Jaśkowski formulated the criterion of construction of an adequate matrix characteristic of intuitionistic logic; the foundations of the theory of models were laid.

[5] M. Tkaczyk, U. Wybraniec-Skardowska, *Polska logika*, op. cit.; J. Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, op. cit.; J. Woleński, *The Achievements of the Polish School of Logic*, in: *The Cambridge History of Philosophy: 1870–1945*, ed. T. Baldwin, Cambridge University Press, Cambridge 2003, pp. 401–416; J. Woleński, *Lvov-Warsaw School*, op. cit.; U. Wybraniec-Skardowska, *The Warsaw School of Logic*, op. cit.; J. Zygmunt, *Polish Logic*, in: *Routledge Encyclopedia of Philosophy*, Vol. 7, ed. E. Craig, Routledge, London–New York 1998, pp. 492–500.

- **Obtaining certain technical results in axiomatization:** The WSL aimed to develop optimal frameworks for propositional calculi, that is, to formulate the smallest possible number of independent and simple axioms using minimal primitive terms; Wajsberg discovered the criterion of finite axiomatizability of such logics; Sobociński and Słupecki obtained seminal results in the area of axiomatization of many-valued logics; Słupecki identified the criterion of definitional fullness of many-valued propositional logics.
- **Creation of three original logical systems by Leśniewski:** Leśniewski, in accordance with his nominalistic views, built three axiomatic systems: protothetic, ontology, and mereology, which he envisioned as a full logical system for the whole of science, including mathematics. By building these systems, with the aim of improving their precision and drawing on the idea of Edmund Husserl's semantic category and the theory of logical types, Leśniewski initiated the theory of semantic/syntactic categories, developed further by Ajdukiewicz, and introduced the differentiation between object language and metalanguage; Sobociński – to a considerable extent – simplified axioms of ontology and protothetic; Tarski, together with Woodger, applied mereology in the axiomatic framework of biology.
- **Formulation of the influential semantic theory of truth by Tarski and defining basic semantic terms:** Tarski, inspired by Aristotle's tradition in philosophy and the non-constructive manner of practising foundations of mathematics, wrote his seminal work on the notion of truth in languages of formalized sciences, first published in Polish (1933), then in German (1936), and later in many other languages. His theory – in its assumptions – constructs a formally correct and substantively apt framework of the classical concept of truth. Tarski also proved, independently of Kurt Gödel, that a set of true expressions of a consistent and sufficiently rich mathematical theory cannot be defined in the language of this theory. To conduct basic reasoning, he introduced the semantic expression of satisfaction of an expression by a sequence of objects; this notion constitutes the basic concept of the theory of models. He also developed, referring to the ideas of Bernard Bolzano, a fundamental definition of logical entailment.

- **Defining and analysing the key concepts of the methodology of deductive sciences:** Tarski and Lindenbaum contributed to work on determining and examining the basic concepts of the methodology of deductive sciences, including the following notions: derivability, consequence operation and deductive theory. Tarski proposed two axiomatic theories of deductive systems based on primitive concepts: a well-formed expression and the consequence operation, and within the theories, he defined the conceptual apparatus concerning the basic properties of deductive systems. Lindenbaum was the author of the so-called theorem of maximalization, which later in the 20th century became one of the most important tools of research on properties of logical systems.
- **Introduction by Jaśkowski of the natural deduction method of the formalization of systems:** Jaśkowski, independently of Gerhard Genzen, introduced a seminal new approach to the formalization of logical systems, the so-called natural deduction method, which offered a non-axiomatic way of characterizing such systems. It referred to the common natural practice of conducting proofs, not only by mathematicians, and, as a very intuitive method, it has been applied in teaching and in computer-based testing of the correctness of theorems proofs.
- **Łukasiewicz's and Słupecki's pioneering research on the axiomatic method of rejection of expressions in juxtaposition with the ordinary axiomatic method:** Łukasiewicz, in his research on Aristotle's syllogistic, introduced the axiomatic method of rejection of expressions. This was possible thanks to his and Słupecki's studies and Słupecki's result, which Łukasiewicz considered to be "the most significant discovery made in the field of syllogistic since Aristotle";[6] the proposed method was complementary to the ordinary axiomatic one, becoming a method of rejection of non-acceptable or false expressions of a system. This represents a dual approach to formalizing deductive systems, on the one hand, as an *assertion system* based on acceptance, and, on the other hand – as a *rejection/refutation system* based on rejection. This method has proven useful for the now-

[6] J. Łukasiewicz, *O sylogistyce Arystotelesa* [On Aristotle's Syllogistic], in: J. Łukasiewicz, *Z zagadnień logiki i filozofii. Pisma wybrane* [Studies in Logic and Philosophy: Selected Writings], ed. J. Słupecki, PWN, Warszawa 1961, p. 226.

popular bi-level characterization of logical systems and the examination of their saturation (Ł-decidability).

- **Łukasiewicz's innovatory research in the field of history of logic:** Łukasiewicz's studies in the history of logic led to the conclusion that the logic of the Stoics was a logic of sentences preceding Aristotle's logic and that it was a logic of "non-provable" forms treated as primitive rules of proving. Łukasiewicz, in studies on Aristotle's syllogistic, discovered that Aristotle's logic was the first axiomatic system (though a non-formalized one). Łukasiewicz formalized the assertoric logic of Aristotle and, together with Słupecki, characterized it also on a second level as a system with respect to rejection/refutation. At the same time, he introduced into science the notion of rejection of expressions itself and a new axiomatic rejection/refutation method. Łukasiewicz's research revolutionized the history of logic, establishing the paradigm of studying it from the point of view of mathematical logic.
- **Research on the foundations of mathematics and set theory:** Within the WSL, important results were obtained as regards elimination of quantifiers and in the sphere of the decidability of certain theories (Tarski, Presburger, Mostowski). Also, significant progress was made concerning the strength and independence of the axiom of choice (Tarski, Lindenbaum, Mostowski).

4. Notable Achievements of Lvov-Warsaw School Logicians Working Outside the Warsaw School of Logic

Logical research within the LWS was not restricted solely to questions of mathematical logic addressed by the WSL.[7] Significant achievements also resulted from, among others, studies in general logical semiotics, which were conducted by Twardowski and his students: Czeżowski, Izydora Dąmbska, Maria Kokoszyńska, Leśniewski, Maria Ossowska, Stanisław Ossowski, Janina Kotarbińska (née Dina Sztejnbarg), and Kotarbiński.

[7] Leon Chwistek also achieved significant results in mathematical logic (theory of logical types). He came from Kraków to Lvov, where he held the Chair of Logic. However, he was not a member of the LWS.

The last of the above-mentioned, after being appointed Chair at the Humanities Department of Warsaw University (1919), established an independent centre in Warsaw, grouping logicians who represented less formal interests in logic and gravitated rather towards logical semiotics and methodology of sciences. Still, that centre maintained close contacts with the WSL and, as a result, the cooperation of the two Warsaw-based centres led to the formation of a division of broadly conceived logic into formal logic, logical semiotics and methodology of sciences.

One of the logicians working outside the WSL, who obtained widely acclaimed, outstanding results in the domain of logical semiotics and methodology of sciences, was undoubtedly Kazimierz Ajdukiewicz, Twardowski's student. His most significant achievements with reference to logic include:

- advancement of the innovative logical theory of questions;
- perfection of Leśniewski's theory of syntactic categories and formation of foundations of categorial grammar (these achievements are regarded as pioneering for so-called mathematical linguistics);
- elaboration of the concept of directival meaning;
- results in logical analysis of the theory of knowledge;
- comprehensive development of issues of definition and classification of reasoning;
- modern formulation of the definitions of entailment, proof and theorem.

Members of the LWS were interested, in particular, in the problem area of the application of formal logic to philosophy. The method of paraphrases elaborated by Ajdukiewicz represented a great achievement in this domain.

Applying the methods of formal logic (axiomatization) to philosophy of nature was investigated by another researcher belonging to the first generation of Twardowski's students – Zygmunt Zawirski, an outstanding representative of the LWS. He is regarded as the originator of new ideas in the methodology of science. During the interwar period, he worked in Kraków and Poznań. He created the first concept of quantum logic in the world, preceding, by a few years, the work of Garrett Birkhoff and John von Neumann.

The first generation of Twardowski's students interested in logic included also Czeżowski. His major achievements in logic are the following: an extended version of Aristotle's syllogistic, original reconstruction of Aristotle's theory of modal propositions, and classification of types of reasoning.

5. The Global Influence of Lvov-Warsaw School Logicians on 20th-Century Logic

The logical research in the LWS resonated broadly in world logic. The activity and outstanding achievements of representatives of the WSL attracted the attention of many promising young logicians. The WSL sometimes achieved unprecedented results even at the Master's thesis level (Wajsberg, Słupecki). Consequently, logic became an attractive subject of study in Warsaw.

In his *Geschichte der Logic*, Heinrich Scholz of Münster remarked: "Warsaw became the main centre of logical studies."[8] Thus, in the lifetime of one generation, "Polish" logic developed from its foundations to achieve international acclaim. Similarly, Abraham A. Fraenkel, Yehoshua Bar-Hillel, and Azriel Levy state the following in their well-known book: "Probably no other country, taking into account the size of its population, has contributed so greatly to the development of mathematical logic and foundations of mathematics as Poland."[9]

What was the lasting contribution of the WSL? The 400-page volume entitled *Polish Logic in 1920–1939* features translations of 17 articles by Polish logicians.[10] All but two are from the WSL. Also in that period, the *Selected Works* of Łukasiewicz appeared in two versions: the first edited by Słupecki (in the Polish language), the second by Ludwik Borkowski (in English), in which we can find a translation of Łukasiewicz's pioneering paper *O logice trójwartościowej* [On Three-Valued Logic] dating from 1920.[11] Let us add that the pioneering research by Łukasiewicz (and Słupecki) on Aristotle's syllogistic was published in the monograph *On Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*.[12] Another key publication was the monograph by E.C. Luschei entitled *The Logical Systems of Leśniewski*.[13] Let us note, too, that all the works by Leśniewski

[8] H. Scholtz, *Geschichte der Logic*, Junker und Dunnkaupt, Berlin 1931, p. 87. Unless stated otherwise, all translations are my own.

[9] A. Fraenkel, Y. Bar-Hillel, A. Levy, *Foundations of Set Theory*, North-Holland Publishing, Amsterdam 1958, p. 200.

[10] S. McCall, ed., *Polish Logic in 1920–1939*, op. cit.

[11] J. Łukasiewicz, *Z zagadnień logiki i filozofii. Pisma wybrane* [From Issues of Logic and Philosophy: Selected Works], ed., introd. J. Słupecki, Państwowe Wydawnictwo Naukowe, Warszawa 1961; J. Łukasiewicz, *Selected Works*, ed. L. Borkowski, North-Holland Publishing, Amsterdam 1970.

[12] J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Clarendon Press, Oxford 1951; 2nd ed. 1957.

[13] E.C. Luschei, *The Logical Systems of Leśniewski*, North-Holland Publishing, Amsterdam 1962.

were translated into English in his *Collected Works*.[14] Tarski's definitive work on the concept of truth, published in Polish in 1933, was translated into many languages: first, in 1936, into German. Its English version under the title *The Concept of Truth in Formalized Languages* was included in the wide selection of Tarski's pre-war articles published as *Logic, Semantics, Mathematics*.[15] It is also worth mentioning that Tarski's important work *What Are Logical Notions?*, published posthumously and edited by John Corcoran, was based on a famous 1936 joint paper of Lindenbaum and Tarski that had previously been relatively little known.[16] Tarski's posthumously edited 1986 paper explains the conception of logical notions; this explanation does not resolve the philosophical problem of the reducibility of mathematical notions to logic.[17] Slightly later, the name Tarski appears in the title of a book by the English philosopher Peter Simons: *Philosophy and Logic in Central Europe from Bolzano to Tarski*, where the author devotes a lot of attention to Tarski and the founders of the WSL – Łukasiewicz and Leśniewski.[18] In turn, Simons's *Parts: A Study in Ontology*, published in Oxford, is an important and impressive contribution to Leśniewski's mereology and contemporary ontology.[19] As one of the reviewers – Timothy Williamson – wrote "*Parts* could be the standard book on mereology for the next twenty or thirty years."[20]

The influence of the leading logician-philosophers of the LWS on the development of logic *sensu largo* and its practising was considerable too. Kotarbiński's first book, *Szkice praktyczne* [Practical Sketches] from 1913, containing ideas of praxeology – the science of efficient action, a new discipline promoted by Kotarbiński – was reprinted in his *Selected Writings* in 1957,[21] and his major work

[14] S. Leśniewski, *Collected Works*, 2 vols., eds. S.J. Surma, J.T. Srzednicki, D.I. Barnet, Wydawnictwo Naukowe PWN, Warsaw; Kluwer, Dordrecht 1992.

[15] A. Tarski, *Logic, Semantics, Metamathematics: Papers from 1926 to 1938*, trans. J.H. Woodger, Clarendon Press, Oxford 1956; A. Tarski, *Logic, Semantics, Metamathematics: Papers from 1926 to 1938*, trans. J.H. Woodger, ed. J. Corcoran, 2nd ed., Hackett Publishing Company, Indianapolis 1983.

[16] It was originally published in German as A. Lindenbaum, A. Tarski, *Über die Beschränktheit der Ausdrucksmittel deduktiver Theorien*, "Ergebnisse eines mathematischen Kolloquiums" 1936, Vol. 7, pp. 15–22.

[17] A. Tarski, *What Are Logical Notions?*, "History and Philosophy of Logic" 1986, Vol. 7, pp. 143–154.

[18] P. Simons, *Philosophy and Logic in Central Europe from Bolzano to Tarski: Selected Essays*, Kluwer, Dordrecht–Boston–London 1992.

[19] P. Simons, *Parts: A Study in Ontology*, Clarendon Press, Oxford 1987.

[20] This sentence appears in the blurb on the cover.

[21] T. Kotarbiński, *Szkice praktyczne. Zagadnienia z filozofii czynu* [Practical Sketches: Problems from Philosophy of Act], in: *Wybór pism* [Selected Writings], Vol. 1, Państwowe Wydawnictwo Naukowe, Warszawa 1957, pp. 13–168; 2nd revised ed. 1961.

Elementy teorii poznania, logiki formalnej i metodologii nauk [Elements of the Theory of Knowledge, Formal Logic and Methodology of Sciences] from 1929,[22] which was the foundation of education for several generations of Polish students and not only students, was translated into English and published in Oxford in 1966, under the title *Gnosiology: The Scientific Approach to the Theory of Knowledge*.[23] Important results obtained by Ajdukiewicz in the field of logical theory of language were published in German and became fairly widely known already before World War II (*Sprache und Sinn* from 1934 and *Die syntaktische Konnexität* from 1935). These and other works dealing with logical semiotics, containing Ajdukiewicz's findings discussed earlier in section 4, including the seminal work on interrogative sentences, found in his valuable booklet under the title *Logiczne podstawy nauczania* [Logical Foundations of Teaching] of 1934, were translated into English, collected and published by Jerzy Giedymin in the volume *The Scientific World-Perspective and Other Essays, 1931–1963*.[24] Ajdukiewicz's 1934 booklet became the basis of his coursebook of logic and methodology of sciences, *Logika pragmatyczna* (published posthumously), and was also translated into English as *Pragmatic Logic* in 1974.[25] Additionally, Ajdukiewicz's significant 1921 habilitation dissertation on the methodology of deductive sciences was translated into English in 1966.[26]

All of the above-mentioned publications in foreign languages had a significant influence on the global development of logic in the past century. They constitute a precious legacy of the LWS and contribute to the cultivation of the School's traditions – not only the logical ones. It should be noted once again that the School earned its worldwide renown primarily thanks to the creativity of the mathematical logicians of the WSL.

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[22] T. Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, Wydawnictwo Zakładu Narodowego im. Ossolińskich, Lwów 1929.

[23] T. Kotarbiński, *Gnosiology: The Scientific Approach to the Theory of Knowledge*, trans. O. Wojtasiewicz, Pergamon, Oxford 1966.

[24] K. Ajdukiewicz, *The Scientific World-Perspective and Other Essays, 1931–1963*, ed. J. Giedymin, Springer, Dordrecht–Boston 1978.

[25] K. Ajdukiewicz, *Logika pragmatyczna*, Państwowe Wydawnictwo Naukowe, Warszawa 1965; K. Ajdukiewicz, *Pragmatic Logic*, trans. O. Wojtasiewicz, Państwowe Wydawnictwo Naukowe, Reidel, Dordrecht–Boston 1974.

[26] K. Ajdukiewicz, *From the Methodology of the Deductive Sciences*, trans. J. Giedymin, "Studia Logica" 1966, Vol. 19, pp. 9–46.

6. Cultivating the Tradition of Polish Logic and Its Fertile Influence on Contemporary Scientific Research

The flourishing of the LWS and its period of greatest prosperity occurred during the interwar period. This was mainly due to its achievements in logic, among others, in connection with the many-valued logics discovered by Łukasiewicz and Tarski's elaboration of the semantic theory of truth. The outbreak of the World War II disrupted the activity of the School and the WSL at their peak moments.

After the war, Polish logic never regained the renown of the WSL, yet the results obtained by its leading members and research trends set by them were and still are developed and expanded, while the traditions handed down by the School are continued and cultivated to this day.

Obviously, the traditions of Polish logic were continued after the war by those of its representatives who had survived (Twardowski and Leśniewski died before 1939; the Lindenbaums, Presburger, and Wajsberg lost their lives during the war) and their students. However, they dispersed throughout the world or settled down in different places in postwar Poland, never forming a uniform research community of logicians any more.

The following representatives of the WSL continued to contribute to popularizing and developing their own and related results obtained in the WSL, despite having emigrated from Poland: Łukasiewicz (Ireland, Dublin), Tarski (United States, Berkeley), Sobociński (United States, Notre Dame), Lejewski (England, Manchester). Tarski established a famous international logical school in California, which drew on the tradition of the WSL. There, together with his students, he developed the model theory. Similarly, Sobociński created his own centre of formal logic and foundations of mathematics in Notre Dame. He also founded the international journal "Notre Dame Journal of Formal Logic," which is still published and appreciated today. Łukasiewicz, in Dublin, recreated and published his monograph on Aristotle's syllogistic and conducted, among others, studies on rejection methods for certain of its logical systems.

Philosophers and logicians from Warsaw, who did not leave Poland, went elsewhere in the country. Jaśkowski, the independent inventor of natural deduction, settled down in Toruń and continued his scientific activity in logic and general mathematics, leaving a rich output to his contemporaries and followers as well as inspiring them to develop new research, including studies in paraconsistent

logics. Jaśkowski was a pioneer of research in this field. In his paper *Rachunek zdań dla systemów dedukcyjnych sprzecznych* [A Propositional Calculus for Inconsistent Deductive Systems], he provided a philosophical justification and formal construction of discursive logic, which was the first system of paraconsistent logic.[27] He developed discursive logic and provided the motivation and the foundations for the development of several paraconsistent logics. Jaśkowski's research on the application of his method of natural deduction contributed to the construction of important systems of natural deduction, by both Polish and foreign authors. A more refined and simplified version of Jaśkowski's natural deduction system was elaborated by Słupecki and Borkowski in 1958. It was used in teaching, in particular in their book *Elements of Mathematical Logic and Set Theory*, which is very practical, offering intuitive theorem proofs.[28] Its additional value is that, at the initiative of Witold Marciszewski, this system was applied to the automatic proving of theorems and to the constructing and formal testing of the correctness of proofs of theorems on the basis of the computer program MIZAR designed by Andrzej Trybulec (presently MIZAR is extensively used by a few hundred researchers all over the world). Jaśkowski's research is continued in the field of natural deduction by Andrzej Indrzejczak, who created a strong international group in Łódź dedicated to this subject matter.

Słupecki, a student of Łukasiewcz, first remained in Lublin, then moved to Wrocław, which, after the war, became one of the leading logical and mathematical centres in Poland, attracting mathematicians, philosophers and logicians. Słupecki continued, popularized and extended the studies of the WSL (mainly the work of Łukasiewicz, Leśniewski and Tarski). In particular, Słupecki's efforts to render Leśniewski's output more systematic and general have made it accessible to a much wider audience. He founded his own logical centre near Wrocław, in Opole. Słupecki generalized the notion of rejection, which had been introduced by Łukasiewicz, promoted the theory of rejected propositions (a theory built over Tarski's theory of deductive systems), and most of all – disseminated the bi-level method of formalization of deductive systems as assertion systems

[27] S. Jaśkowski, *Rachunek zdań dla systemów dedukcyjnych sprzecznych* [A Propositional Calculus for Inconsistent Deductive Systems], "Studia Societatis Toruniensis, Sec. A" 1948, Vol. 1, pp. 57–77; reprinted in "Studia Logica" 1969, Vol. 24, pp. 143–157.

[28] J. Słupecki, L. Borkowski, *Elements of Mathematical Logic and Set Theory*, Pergamon Press, Oxford 1967.

and rejection/refutation systems, along with testing their Ł-decidability (saturation), which is widely used in the logical world today.

Mostowski, who after the war stayed in Warsaw, created a new centre of mathematical logic and foundations of mathematics, continuing Tarski's work in areas such as set theory, model theory, decidability, and algebraic and topological methods in logic. Members of this centre included outstanding logicians, such as Helena Rasiowa and Andrzej Grzegorczyk. Mostowski and Rasiowa trained a numerous group of logicians and mathematicians, who maintained the renown of the LWS. The above-mentioned representatives of the new Warsaw school and their students achieved results that garnered international acclaim and are frequently cited in the literature on the subject today. Moreover, Grzegorczyk was probably the most outstanding continuator of the LWS tradition, as a logician, mathematician and philosopher-ethicist. He was also a great organizer of international events dedicated to logic and mathematics. He established, among others, the Logical Semester at the Stefan Banach International Mathematical Center in Warsaw, where papers were delivered by several dozen outstanding scholars from all over the world. In 1995, to celebrate the 100th anniversary of the founding of the LWS, he organized a huge international conference: first held in Lvov and soon afterwards in Warsaw. The meetings initiated and arranged by him contributed not only to international integration of the logical community, but also to preserving, promoting and cultivating the logical output of the LWS. This had a prominent impact on international logical literature.

Polish logic and philosophy after the war owed much to Kotarbiński and Ajdukiewicz, who continued their teaching and research: Kotarbiński – in Łódź and then in Warsaw, Ajdukiewicz – first in Poznań and then in Warsaw. Zawirski moved to Kraków, where he continued his scholarly and organizational activities until his untimely death in 1948. Czeżowski settled down in Toruń, where he taught logic (among other courses) and where he founded a strong logical and philosophical centre, which is still active today. He was also the initiator and organizer of a series of international conferences on the history of logic, still regularly taking place in Kraków (now under the name "Cracow Logic Conference").

Kotarbiński initiated and developed a new discipline – praxeology. Ajdukiewicz's work allowed him to advance his earlier logical ideas, which had begun before the war, and opened new avenues for studying questions and answers in modern logic, categorial grammar, and the concept of meaning.

The specific interest of Polish logicians in questions is due to Tadeusz Kubiński, who was the first to develop formally the idea of Ajdukiewicz's questions. His monograph in Polish *Wstęp do logicznej teorii pytań* [An Introduction to the Logical Theory of Questions] appeared in 1971, well ahead of Nuel D. Belnap and Thomas B. Steel's 1976 *The Logic of Questions and Answers*.[29] Kubiński's book was published in English as *Outline of the Logical Theory of Questions* in 1980. Andrzej Wiśniewski, a student of Kubiński from Poznań, developed the concept of inferential erotetic logic in his books and a series of papers published in international journals. Wiśniewski and his students, mainly Dorota Leszczyńska-Jasion, are working on applying inferential erotetic logic in proof theory and in problem-solving.

The pioneering studies conducted by Ajdukiewicz on perfecting Leśniewski's theory of syntactic categories and categorial grammar – the notion coined by Bar-Hillel in his works dating from the 1950s and the 1960s – in the 1980s gained great popularity. In 1985, they were used in my own habilitation dissertation, translated into English as *Theory of Language Syntax: Categorial Approach*.[30] In 1988, the book *Categorial Grammar*, edited by Johan van Benthem, Wojciech Buszkowski, and Witold Marciszewski appeared.[31] These studies were and are currently being continued in Poland by Buszkowski and his students in Poznań.

Ajdukiewicz's ideas on meaning were taken up by me in 2007[32] and became the subject of a research programme conducted by Ryszard Wójcicki. Presently, Janusz Maciaszek is the best-known Polish researcher investigating Ajdukiewicz's concept of derivative meaning and a leading advocate of this approach.

In 1953, Ajdukiewicz also founded the first postwar edition of "Studia Logica," which is now a renowned international journal published by Springer.[33] Wójcicki, as its longtime editor-in-chief, significantly contributed to the journal's international reputation.

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[29] N.D. Belnap, T.B. Steel, *The Logic of Questions and Answers*, Yale University Press, New Haven 1976.

[30] U. Wybraniec-Skardowska, *Theory of Language Syntax: Categorial Approach*, Kluwer Academic Publishers, Dordrecht–Boston 1991.

[31] J. van Benthem, W. Buszkowski, W. Marciszewski, eds., *Categorial Grammar*, John Benjamins Publishing Company, Amsterdam–Philadelphia 1988.

[32] U. Wybraniec-Skardowska, *Logic–Language–Ontology*, Springer, Birkhäuser, Cham 2022, chapter 5.

[33] The first and, at the same time, the only pre-war issue of the journal "Studia Logica," founded by Łukasiewicz, appeared in 1934.

Roman Suszko was another outstanding logician and philosopher based in Warsaw (a student of Zawirski and Ajdukiewicz). He invented non-Fregean logic and, together with Wójcicki, created an active logical centre at the Polish Academy of Sciences in the late 1960s. Thanks to Wójcicki, the centre not only cultivated certain logical traditions of the LWS for many years, but also conceived and carried out new research projects, as well as hosted or trained numerous Polish and foreign logicians known to a greater or lesser degree both in Warsaw and other centres in the country. Wójcicki also organized international conferences known as "Trends in Logic," with publications under the same title. These conferences are still held today. All the above-mentioned activities and efforts have clearly contributed to strengthening the position of Polish logic in international circles.

The activities of the Warsaw-based centre mentioned above and that of several other post-war Polish logical groups ceased to be continued, yet the traditions of the LWS are still alive. Polish logic and philosophy, as well as international scholarship, draw extensively from the heritage of this School.[34]

Polish logic continues to function with vigour today, strengthening its position, developing dynamically, and gaining international renown. A significant share in this recognition of contemporary Polish logicians comes from well-known philosophers: Jan Woleński, Jacek J. Jadacki, and Anna Brożek, who preserve and cultivate the traditions of the famous School, including its logical heritage, through relevant publications, organization of scientific events, and historical-scientific research.[35]

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[34] The book *The Lvov-Warsaw School: Past and Present*, edited by A. Garrido and U. Wybraniec-Skardowska (Springer, Birkhäuser, Cham 2018), contains an extensive discussion of this legacy, with a contribution presenting the main representatives of the School and their works. It includes not only biographies of almost all the leaders of the LWS, but also articles related to their outputs. All the biographies and articles were prepared by Polish and foreign experts on the achievements of the LWS representatives.

[35] I would like to note that Woleński has offered a detailed presentation of the history and achievements of the LWS in his *Filozoficzna Szkoła Lwowsko-Warszawska* and *Logic and Philosophy in the Lvov-Warsaw School*, and in his many other publications, or on the occasion of scientific events devoted to the LWS or its prominent members; he often emphasized the significant role played by logic. In a similar way, Jadacki is known to be an outstanding propagator of the LWS tradition as well as author or/and editor of publications presenting the influence of the LWS and its continuation (see, e.g., his *The Lvov-Warsaw School and Its Influence on Polish Philosophy of the Second Half of 20th Century*, in: *The Lvov-Warsaw School: The New Generation*, eds. J. Jadacki, J. Paśniczek, Rodopi, Amsterdam–New York 2006, pp. 41–83; and *Polish Analytical*

Finally, it should be noted that many Polish logicians of the new generation have found new areas of interest in mathematical and philosophical logic and their applications, in particular in computer science – a field that did not exist at all in the pre-war period.

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Philosophy, Semper, Warsaw 2009). He is also the editor of Leśniewski's original publications and an initiator of commemorating outstanding logicians of the School at the University of Warsaw by naming lecture rooms after individual scholars, marked by placing commemorative plaques at the entrance to the halls. Anna Brożek, a student of Jadacki, has continued the activity of her mentor. Presently, she heads the Lvov-Warsaw School Research Center established at the University of Warsaw, which has flourished under her excellent leadership.

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