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On the Polish "Via Modalization" Approach to Paraconsistency

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Abstract: In this paper, I survey the history of the Polish tradition of paraconsistency and its chronological development. I outline the features of this tradition to provide some insights into the more general notion of philosophical schools. The main features of the Polish tradition of paraconsistency are the continuation of research on a previous philosophical tradition and international collaboration.

Key words: Polish tradition, discussive logic, paraconsistent, modal logic, inconsistency

1. Introduction

This paper is devoted to revisiting the Polish approach to paraconsistent logic developed by the Polish logician Professor Stanisław Jaśkowski. In 1948 –seventy-five years ago – Jaśkowski published his revolutionary paper *A Propositional Calculus for Inconsistent Deductive Systems*,[1] dealing with the problem of:

1) providing a calculus for inconsistent systems that do not entail its triviality (or *overfilling*),
2) which would be rich enough to allow for practical inferences, and
3) which would have intuitive justification.

Having critically assessed several options to solve this problem, Jaśkowski proposed a logical system which was named $\mathbf{D}_2$ after two-valued discussive sentential calculus. Part of its motivation was to allow for models of a discussion where participants contradict one another.

[1] S. Jaśkowski, *Rachunek zdań dla systemów dedukcyjnych sprzecznych*, "Studia Societatis Scientiarum Torunensis" 1948, Vol. 1, No. 5, pp. 55–77; S. Jaśkowski, *A Propositional Calculus for Inconsistent Deductive Systems*, "Logic and Logical Philosophy" 2004, Vol. 7, pp. 35–56.

Some scholars found further applications of discussive logic for the following tasks:[2]

– modelling views that accept true contradictions,
– representing systems of hypotheses that contradict established laws of science,
– handling vague terms and imprecise concepts.

In this paper, I investigate the essential features of the Polish approach to paraconsistency, and the morals that one can obtain from it to outline a conception of a "philosophical school." More specifically, I seek to determine the model of the Polish tradition of paraconsistency, which was initiated by Jaśkowski, reinstated in Toruń by Jerzy Perzanowski, and which has been kept alive since then by Andrzej Pietruszczak, Marek Nasieniewski, and Krystyna Mruczek-Nasieniewska.

There is a vast well-known survey on the history of discussive logic.[3] In most cases, however, not much attention has been paid to the chronology of works developed after Jaśkowski's paper and the evolution of his conceptual insights. The aim of this study is thus to contribute to both aspects. I would like to distinguish the historical stages of the Polish tradition of paraconsistency and reflect on its evolution. In the paper, I do not look at the full formal details of discussive logic but emphasize the historical and intuitive aspects instead.

The paper is divided into three sections. In the first part, I describe the life and work of Stanisław Jaśkowski and focus particularly on his research of the discussive logic $\mathbf{D_2}$. In this section, while describing his life and work, I follow standard literature as well as some facts not found elsewhere (to the best of the author's knowledge). In the second section, I describe the development of discussive logic

[2] N.C.A. da Costa, L. Dubikajtis, *On Jaśkowski's Discussive Logic*, in: *Non-Classical Logics, Model Theory and Computability : Proceedings of the Third Latin-American Symposium on Mathematical Logic, Campinas, Brazil, July 11–17, 1976*, eds. A. Arruda, N.C.A. da Costa, R. Chuaqui, North-Holland Publishing Company, Amsterdam–New York–Oxford 1977, pp. 37–56; J. Kotas, *Discussive Sentential Calculus of Jaśkowski*, "Studia Logica" 1975, Vol. 34, No. 2, pp. 149–168; J. Kotas, N.C.A. da Costa, *A New Formulation of Discussive Logic*, "Studia Logica" 1979, Vol. 38, No. 4, pp. 429–445.

[3] For notable examples, see L. Dubikajtis, *The Life and Works of Stanisław Jaśkowski*, "Studia Logica" 1975, Vol. 34, No. 2, pp. 109–116; J. Kotas, A. Pieczkowski, *Scientific Works of Stanisław Jaśkowski*, "Studia Logica" 1967, Vol. 21, pp. 7–15; A.I. Arruda, *Aspects of the Historical Development of Paraconsistent Logic*, in: *Paraconsistent Logic: Essays on the Inconsistent*, eds. G. Priest, R. Routley, J. Norman, Philosophia Verlag, München–Hamden–Wien 1989, pp. 99–130; J. Ciuciura, *History and Development of the Discursive Logic*, "Logica Trianguli" 1999, Vol. 3, pp. 3–31.

after Jaśkowski. In the third section, I sketch some general criteria to determine the notion of the philosophical "school" by analyzing the features of the Polish tradition of paraconsistency, and discuss the prospect of its future.

2. Life and Work of Stanisław Jaśkowski

Stanisław Jaśkowski (1906–1965) was a Polish mathematician working under the supervision of the logician and philosopher Jan Łukasiewicz.[4] Jaśkowski studied mathematics at the University of Warsaw, where he later obtained a PhD in philosophy.

Jaśkowski's scientific research was mainly concerned with mathematical logic, geometry, and arithmetic.[5] In the field of mathematical logic, he made important contributions to paraconsistent logic, natural deduction,[6] intuitionistic logic, free logic, and decidability. In geometry and arithmetic, he contributed to the geometry of solids, foundations of geometry, the notion of symmetry and ornaments, and the notion of number. According to Kotas and Pieczkowski, the research of Jaśkowski on mathematical logic was characterized by two main topics:[7]

1) (un)decidability of various systems, and
2) foundations of geometry.

As a former student of Łukasiewicz, Jaśkowski was influenced by Łukasiewicz's research on the principle of non-contradiction.[8] Such an influence is also present in his PhD dissertation,[9] where he undertook the effort of answering a problem

4 Jan Woleński in *Lvov-Warsaw School*, in: *The Stanford Encyclopedia of Philosophy* (Summer 2022 Edition), ed. E.N. Zalta, URL: https://plato.stanford.edu/entries/lvov-warsaw/ (susbstantive revision on 30.09.2019), observed that Jan Łukasiewicz and Stanisław Leśniewski were philosophers with modest mathematical backgrounds invited to the Faculty of Mathematical and Natural Sciences of the University of Warsaw to teach mathematical logic. It was not strange that a philosopher was a mentor of a mathematician. Jaśkowski was also a student of Leśniewski and of Alfred Tarski (see L. Dubikajtis, *The Life and Works of Stanisław Jaśkowski*, op. cit., p. 109).

5 J. Kotas, A. Pieczkowski, *Scientific Works of Stanisław Jaśkowski*, op. cit.

6 A. Indrzejczak, *Powstanie i ewolucja dedukcji naturalnej*, "Filozofia Nauki" 2014, Vol. 22, No. 2, pp. 5–19.

7 J. Kotas, A. Pieczkowski, *Scientific Works of Stanisław Jaśkowski*, op. cit.

8 J. Łukasiewicz, *O zasadzie sprzeczności u Arystotelesa*, Akademia Umiejętności, Fundusz Wydawniczy im. W. Osławskiego, Kraków 1910.

9 S. Jaśkowski, *On the Rules of Suppositions in Formal Logic*, Seminarjum Filozoficzne Wydziału Matematyczno-Przyrodniczego Uniwersystetu Warszawskiego, Warszawa 1934.

raised by Łukasiewicz in 1925,[10] to wit: expressing formally the way that mathematicians actually reason and carry out their proofs. As Łukasiewicz noted, mathematicians reason without appealing to the theses of the theory of deduction, and instead they proceed by making suppositions.[11]

The outbreak of the Second World War caused Jaśkowski to not obtain his habilitation. He instead volunteered to defend Warsaw. At the time, most of his scientific works were destroyed, and he had to rewrite them from memory. After the Second World War, he lectured at the University of Łódz, and later moved to Toruń in 1945.[12] Then, he obtained his habilitation in Kraków under the supervision of Zygmunt Zawirski with a dissertation on real numbers.[13] He organized the Faculty of Mathematics, Physics, and Chemistry, and later became its dean. Then, he became vice-rector, and rector of the Nicolaus Copernicus University in Toruń. It is important to remark that as a consequence of the war, the Nicolaus Copernicus University was lacking scientific staff. Jaśkowski had to then take charge of several courses and seminars in mathematical logic, probability, and set theory.[14]

Jaśkowski also worked as a collaborator of the "Journal of Symbolic Logic," and as joint editor of "Studia Logica" and "Zeitschrift für Matematische Logik und Grundlagen der Mathematik." After the Second World War, his publications summed up to forty-seven contributions, including scientific works, reviews, reports, and lectures.[15]

His social activity was marked by modernizing the programmes of mathematics in secondary schools. For this reason, he was concerned with the way

[10] See S. Jaśkowski, *Elementy logiki matematycznej i metodologii nauk ścisłych*, ed. A. Indrzejczak, Wydawnictwo Uniwersytetu Łódzkiego, Łódz 2018, p. x.

[11] As it has been emphasized in J. Kotas, A. Pieczkowski, *Scientific Works of Stanisław Jaśkowski*, op. cit., pp. 7–15; S. Jaśkowski, *Elementy logiki matematycznej i metodologii nauk ścisłych*, op. cit., Gerhard Gentzen is mostly recognized as the forerunner of natural deduction systems, even though Jaśkowski investigated the topic eight years earlier at a seminar imparted by Łukasiewicz (see G. Gentzen, *Untersuchungen über das logische Schließen. I & II*, "Mathematische Zeitschrift" 1934, Vol. 39, pp. 176–210, for more information on Gentzen's work). The reason for this situation was the delay of the publication of his doctoral dissertation for eight years, due to health problems (see S. Jaśkowski, *Elementy logiki matematycznej i metodologii nauk ścisłych*, op. cit., p. x).

[12] At the time, Tadeusz Czeżowski, also a student of Łukasiewicz and an important member of the Lvov-Warsaw School, also moved to Toruń. However, Czeżowski went to the department of philosophy at Nicolaus Copernicus University. Czeżowski, however, did not collaborate with Jaśkowski there.

[13] S. Jaśkowski, *Elementy logiki matematycznej i metodologii nauk ścisłych*, op. cit., p. xiv.

[14] Ibid., p. xii.

[15] L. Dubikajtis, *The Life and Works of Stanisław Jaśkowski*, op. cit., p. 110.

that mathematics is taught, and devoted much of his time to improving the programmes at secondary schools.

The legacy of Jaśkowski spread through his four students, who continued to develop the scientific interest of their mentor. These students were Lech Dubikajtis, Jerzy Kotas, August Pieczkowski, and Aleksander Ciopa-Śniatycki.[16] Jaśkowski planned to work with each of them, separately, on the topics of discussive logic, decision procedures, natural deduction, and causal functions. However, his students were interested in all of the different topics, and sometimes contributed to more than one of them.

In the rest of the paper, I will focus on describing Jaśkowski's "discussive logic," a paraconsistent logic inspired by an analysis of discussions, by far the most recognized contribution of Jaśkowski in the global logical community. It is significant to note, however, that Jaśkowski did not find himself primarily interested in this topic, but in the research on causal functions. It was the case that paraconsistent logic became studied in several places in the world, and, as a consequence, Jaśkowski's research on discussive logic received more attention than his investigation on causal functions. As a matter of fact, Jaśkowski's research led in some way to the big development of logic made sometime later in Brazil with Professor Newton Carneiro Affonso da Costa.

3. The Discussive Logic D_2

Consider a formal language, *L*, and a formula, *A* of *L*. A paraconsistent logic is a logic where a contradiction does not imply an arbitrary formula. More specifically, a logic, **L**, is paraconsistent if and only if the principle *ex contradictione sequitur quodlibet*, that from a contradiction any conclusion follows, that is, for any *A and B*:

$$A, \sim A \vDash B,$$

is not valid.[17] Discussive logic is a paraconsistent logic in which one can represent opposing opinions from a discussion. Jaśkowski considered the possibil-

[16] I am grateful to Bogumiła Maria Klemp-Dyczek for providing me the reference of Aleksander Ciopa-Śniatycki.

[17] See I.M.L. D'Ottaviano, E.L. Gomes, *Gerland's Dialectica and Paraconsistency*, "Edukacja Filozoficzna" 2021, Vol. 70, pp. 143–170, for a detailed discussion of the conception of paraconsistent logic.

ity of adding a diamond connective, $\Diamond$, in front of a formula, A, to show how an impartial arbiter should evaluate the assertions made in a discussion. Thus, when someone in the discussion says A, an impartial arbiter must consider this assertion as "only possible." To model such discussions, Jaśkowski used the modal logic **S5**, as it permits to represent the external observer as aware of all the assertions made in the discussion by every participant. In discussive logic, all participants are aware of all the assertions of all other participants. According to this point of view, a participant in a discussion can assert a particular formula A, say $\Diamond A$–, another participant can assert its negation $\sim A$, say $\Diamond \sim A$, but neither of the two, nor the observer themself, needs to assert any unrelated formula B, say $\Diamond B$. But this is nothing more than invalidating the principle of explosion, thus making $\mathbf{D}_2$ a paraconsistent logic, due to the fact that $\Diamond A, \Diamond \sim A \nvDash \Diamond B$ on the basis of **S5**. Furthermore, following the previous intuition, a participant in a discussion can assert A ($\Diamond A$) another participant can assert B ($\Diamond B$) but neither of the two needs to assert its conjunction $A \wedge B$ ($\Diamond(A \wedge B)$) where $\wedge$ is classical. Thus, one arrives at invalidating the principle of adjunction:

$$A, B \nvDash (A \wedge B).$$

After considering adding the possibility connective in front of assertions, Jaśkowski introduced the logic $\mathbf{D}_2$ using some "discussive" language with a discussive implication, $\rightarrow_d$, and a (right) discussive conjunction, $\wedge^r_d$ as a way to express the idea conveyed by the use of the possibility.[18] In intuitive terms, a formula with the form $A \rightarrow_d B$ is interpreted as "if some participant asserts A in the discussion, then B." Correspondingly, a formula with the form $A \wedge^r_d B$ is interpreted as "A, and some participant asserts B in the discussion." As it has been noted after Jaśkowski,[19] another theoretical possibility is to interpret the discussive conjunction as saying: "some participant asserts A, and also B," where the first conjunct is "modalized" instead of the second one. For this second conjunction, one can use the (left) discussive conjunction $\wedge^l_d$ to build formulas with the form $A \wedge^l_d B$, with the previous intended interpretation.

To be sure, the assertions made in a discussion can be used to draw inferences. As such, the assertions are merely considered possibly true from the point of view

[18] In fact, in his original paper Jaśkowski used Polish notation.
[19] N.C.A. da Costa, L. Dubikajtis, *On Jaśkowski's Discussive Logic*, op. cit.

of some external observer of the discussion,[20] where the observer has access to the assertions made by each participant of the discussion. Thus, if some participant d_1 asserts A, and another participant d_2 asserts B, then the external observer has among his records both $\Diamond A$ and $\Diamond B$, from which he can, for instance, conclude either $\Diamond A$ or $\Diamond B$. This example shows that the premises and conclusions made in a prototypical discussion are taken by its participants under proviso, that is – in a metaphoric way – that they are always preceded by the symbol $\Diamond$.

One can consider the inference $A \vDash_d B$ to be valid in the discussive logic $\mathbf{D_2}$ just in case for all A and B, $\Diamond \mathrm{tr}(A) \vDash \Diamond \mathrm{tr}(B)$ is valid in the modal logic **S5**. Here tr(x) is the respective translation of discussive formulas into the modal language according to their above-sketched meanings.

4. The Development of Discussive Logic

After the introduction of the discussive logic $\mathbf{D_2}$, logicians began to explore all its potential. This was done thanks to the efforts of Jaśkowski's students Lech Dubikajtis and Jerzy Kotas.

In 1967, Dubikajtis met Newton da Costa in Paris and the two started to work together. Just one year later, Dubikajtis and da Costa published the paper *Sur la logique discursive de Jaśkowski*,[21] where they discussed the first axiomatization of $\mathbf{D_2}$, that is, providing a set of formulas from which all the formulas that are valid in discussive logic can be derived. In that work, Dubikajtis and da Costa also distinguished between two kinds of languages for the axiomatization of $\mathbf{D_2}$: a propositional language with the discussive connectives $\rightarrow_d$, and $\wedge^r_d$, and a modal language with the connectives $\Diamond$ and $\Box$ but without the discussive connectives. Later, Dubikajtis and his students Grażyna Achtelik, Elżbieta Dudek, and Jan Konior investigated in two articles another axiomatization (this axiomatization was introduced by da Costa and Dubikajtis, as I will explain below).[22] Dubikajtis and

20 See K. Mruczek-Nasieniewska, M. Nasieniewski, A. Pietruszczak, *A Modal Extension of Jaśkowski's Discussive Logic D_2*, "Logic Journal of the IGPL" 2019, Vol. 27, p. 451.

21 N.C.A. da Costa, L. Dubikajtis, *Sur la logique discursive de Jaśkowski*, "Bulletin de L'Académie Polonaise des Sciences" 1968, Vol. 16, No. 7, pp. 551–557.

22 G. Achtelik, L. Dubikajtis, E. Dudek, J. Konior, *On Independence of Axioms in Jaśkowski Discussive Propositional Calculus*, "Reports on Mathematical Logic" 1981, Vol. 11, pp. 3–11; L. Dubikajtis, E. Dudek, J. Konior, *On Axiomatics of Jaśkowski's Discussive Propositional Calculus*, in:

Kotas also were invited teachers, and spent some periods in Brazil, working with da Costa at the University of Campinas (UNICAMP, State of São Paulo).[23] The research on discussive logic in Poland was continued also by Kotas and his students Tomasz Furmanowski, Wiesław Dziobiak, Jerzy Błaszczuk, and Max Urchs.

I propose to divide the kind of research on discussive logic made after the meeting of Dubikajtis and da Costa into the following topics:

- axiomatization of the discussive logic $\mathbf{D}_2$ and different proof-theoretic presentations of $\mathbf{D}_2$,
- algebraization of $\mathbf{D}_2$ and mathematical theories relying on $\mathbf{D}_2$,
- modal logic counterparts that can define $\mathbf{D}_2$ and other discussive systems,
- generalization of discussive logics: discussive consequence and discussive negation, and
- philosophical view on discussive logics, and relation with other paraconsistent logics.

In 1968, Kotas investigated which algebra underlies the system $\mathbf{D}_2$ as a mathematical theory that corresponds to a given logical system.[24] The research on the mathematical dimension of discussive logic was then transported to Brazil. In 1970, Lafayette de Moraes wrote, under the supervision of da Costa, the first thesis on discussive logic.[25] This thesis was devoted to presenting a first-order discussive logic different from the one introduced by da Costa and Dubikajtis in 1968. In the thesis, de Moraes also discussed the prospects of a discussive set theory based on this system of logic. The same year, da Costa and Itala M. Loffredo D'Ottaviano introduced the system **J3**,[26] a modal three-valued paraconsistent logic, as a solution to the problem proposed by Jaśkowski. In 1973, de Moraes wrote his PhD dissertation, *Lógica discursive e modelos de Kripke* [Discussive Logic and Kripke Models], also under the supervision of da Costa, where de Moraes introduced

Proceedings of the Third Brazilian Conference on Mathematical Logic, eds. A.I. Arruda, N.C.A. da Costa, A.M. Sette, Sociedade Brasileira de Lógica, São Paulo 1980, pp. 109–117.

23 I thank an anonymous reviewer for this observation.

24 N.C.A. da Costa, L. Dubikajtis, *Sur la logique discursive de Jaśkowski*, op. cit., pp. 551–557.

25 L. de Moraes, *Sobre a lógica discursiva de Jaśkowski*, master's thesis, University of São Paulo, 1970.

26 I.M.L. D'Ottaviano, N.C.A. da Costa, *Sur un problème de Jaśkowski*, "Comptes Rendus de l'Académie des Sciences" 1970, Vol. 270, pp. 1349–1353.

an axiomatic system for discussive logic in the modal language, and a Kripke semantics for it.[27]

In 1974, Kotas showed that the discussive system is finitely axiomatizable. To put it roughly, the idea is that one can provide a finite set of formulas from which all the formulas that are valid in $\mathbf{D_2}$ can be derived.[28] In the same year, Kotas showed that the discussive logic is characterized by an infinite quantity of values.[29] One year later, in 1975, Furmanowski showed that one can use any modal system **M** intermediate between the modal systems **S4** and **S5** for the translation, tr(x), of the discussive formulas.[30] This became a significant discovery, as one could consider other modal systems aside from **S5** as a basis for the discussive logic $\mathbf{D_2}$. To put it differently, the full modal system **S5** is not necessary to obtain the discussive logic $\mathbf{D_2}$.

The $\Diamond^n$-counterpart of a modal system **M** is the set of formulas that are valid after preceding them with the symbol $\Diamond$ n times: $\mathbf{M} \vDash \Diamond \ldots \Diamond B$; the $\Box^n$-counterpart of a modal system **M** is the set of formulas that are valid after preceding them with the symbol $\Box$ n times: $\mathbf{M} \vDash \Box \ldots \Box\ B$. The $\Diamond^n$-counterpart of the modal system **S5** can be used to characterize the discussive logic $\mathbf{D_2}$ via the translation function tr(x). In 1975, Perzanowski discussed the $\Box^n$-counterparts and $\Diamond^n$-counterparts of different modal systems other than **S5**, and considered various modal systems inspired by the way of obtaining discussive logic.[31] In 1976 Błaszczuk and Dziobiak investigated the problem of the axiomatization of $\Diamond^n$-counterparts of various modal systems.[32]

In 1977, da Costa and Dubikajtis replaced Jaśkowki's discussive conjunction with one where the translation of the discussive conjunction "possibilitates" the

27 See E.H. Alves, A.E. Consalvo, *Contribuiões do Professor Lafayette de Moraes para o Desenvolvimiento da Lógica Matemática no Brasil (Contributions by Professor Lafayette de Moraes to the Development of Mathematical Logic in Brazil)*, "Cognitio" 2009, Vol. 10, No. 2, pp. 185–190.

28 J. Kotas, *The Axiomatization of Stanisław Jaśkowski's Discussive System*, "Studia Logica" 1974, Vol. 33, No. 2, pp. 195–200.

29 J. Kotas, *On Quantity of Logical Values in the Discussive D_2 System and in Modular Logic*, "Studia Logica" 1974, Vol. 33, No. 3, pp. 273–275.

30 T. Furmanowski, *Remarks on Discussive Propositional Calculus*, "Bulletin of the Section of Logic" 1975, Vol. 4, No. 1, pp. 33–36.

31 J. Perzanowski, *On M-Fragments and L-Fragments of Normal Modal Propositional Logics*, "Reports on Mathematical Logic" 1975, Vol. 5, pp. 63–72.

32 J.J. Błaszczuk, W. Dziobiak, *An Axiomatization of M^n-Counterparts for Some Modal Logics*, "Reports on Mathematical Logic" 1976, Vol. 6, pp. 3–6.

first conjunct, namely, $f(A \wedge^{1}_{d} B) = \Diamond f(A) \wedge f(B)$.[33] Furthermore, they extended this new system and axiomatized it. In the same year, Błaszczuk and Dziobiak[34] studied the ◊-counterparts of modal systems, including Sobociński's ones, as some of them can be used to define the logic $\mathbf{D_2}$. Also in this year, Kotas and da Costa investigated ◊-counterparts of various families of modal systems, including minimal ones that could be used to characterize the discussive logic.[35]

In 1979, Kotas and da Costa introduced a natural deduction system for the discussive logic with the (left) discussive conjunction $A \wedge^{1}_{d} B$.[36] The next year, Dubikajtis, Dudek and Konior investigated da Costa and Dubikajtis's work of 1977.[37] They showed the dependence of the axioms that do not use negation and then reduced them. In 1978, Kotas and da Costa offered a solution to Jaśkowski's problem of providing a calculus for inconsistent systems that do not entail its triviality by using Łukasiewicz's many-valued logics.[38] In 1981, Achtelik, Dubikajtis, Dudek and Konior showed the dependence of the axioms that use negation in da Costa and Dubikajtis's work of 1977 and also reduced them.[39]

An M-counterpart of a modal system can be also treated more generally as the set of formulas that are valid after preceding them with arbitrary successions of ◊, □ or ~: $\vDash \sim\Diamond\ldots\Box(B)$. In 1984, Błaszczuk investigated the M-counterparts (or "M-extensions") of various modal systems, where the notion of M-counterparts was meant in a more general way.[40] By taking the M-counterparts of a given modal system – instead of its ◊-counterparts – one could obtain various logical systems, some of which can be considered discussive.

33 N.C.A. da Costa, L. Dubikajtis, *On Jaśkowski's Discussive Logic*, op. cit.

34 J.J. Błaszczuk, W. Dziobiak, *Modal Logics Connected with Systems S4$_n$ of Sobociński*, "Studia Logica" 1977, Vol. 36, pp. 151–164.

35 J. Kotas, N.C.A. da Costa, *On Some Modal Logical Systems Defined in Connexion with Jaśkowski's Problem*, in: *Non-Classical Logics, Model Theory and Computability: Proceedings of the Third Latin-American Symposium on Mathematical Logic, Campinas, Brazil, July 11–17, 1976*, eds. A. Arruda, N.C.A. da Costa, R. Chuaqui, North-Holland Publishing Company, Amsterdam–New York–Oxford 1977, pp. 57–73.

36 J. Kotas, N.C.A. da Costa, *A New Formulation of Discussive Logic*, op. cit.

37 L. Dubikajtis, E. Dudek, J. Konior, *On Axiomatics of Jaśkowski's Discussive Propositional Calculus*, op. cit., pp. 109–117.

38 J. Kotas, N.C.A. da Costa, *On the Problem of Jaśkowski and the Logics of Łukasiewicz*, in: *Proceedings of the First Brazilian Conference*, eds. A.I. Arruda et al., Marcel Dekker, New York 1978, pp. 127–139.

39 G. Achtelik, L. Dubikajtis, E. Dudek, J. Konior, *On Independence of Axioms in Jaśkowski Discussive Propositional Calculus*, op. cit., pp. 3–11.

40 J.J. Błaszczuk, *Some Paraconsistent Sentential Calculi*, "Studia Logica" 1984, Vol. 43, pp. 51–61.

In 1985, de Moraes introduced a "discussive" set theory based on a first-order logic with equality using the modal system **S5**.[41] One year later, 1986 Urchs defined a system for a discussive logic that can also be used to represent causal relations.[42] It thus combines Jaśkowski's two central interests in mathematical logic.

In 1989, Kotas and da Costa presented some open problems on the discussive logic $\mathbf{D}_2$: whether certain discussive systems based on various modal logics are axiomatizable or not, the algebraization of various modal systems, and modal logics based on different non-classical logics.[43] This constitutes an important summary on discussive logic done so far. In 1995, da Costa and Francisco Doria discussed the idea of pragmatic truth and perspectives on the foundation of physics in the context of discussive logic.[44]

In 1998, in Toruń, the Memorial Symposium "Paraconsistent Logic, Logical Philosophy, Informatics and Mathematics" was organized on the occasion of the anniversary of Jaśkowski's seminal paper. During this event, the Medal of Merit "Nicolaus Copernicus" was awarded to Newton da Costa; the University of Toruń awarded him this Medal due to his important contributions. In the international journal "Logic and Logical Philosophy," a new translation of Jaśkowski's seminal paper with notes by Jerzy Perzanowski was published.

In a 2001 article, de Moraes and Jair Minoro Abe presented a discussive logic of zeroth order.[45] They axiomatized this logic, and showed that the resulting logic was different from the logic introduced by da Costa and Dubikajtis in 1977.

In 2002, Urchs challenged the conception of paraconsistent logic.[46] A paraconsistent logic had been characterized as a logic where it is not true that from an inconsistency anything follows. Urchs held that this definition is derivative from

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41 L. de Moraes, *On Discussive Set Theory*, "Bulletin of the Section of Logic" 1985, Vol. 14, No. 4, pp. 144–148.

42 M. Urchs, *On Two Systems of Stanisław Jaśkowski*, "The Journal of Non-Classical Logic" 1986, Vol. 3, No. 1, pp. 25–32.

43 J. Kotas, N.C.A. da Costa, *Problems of Modal and Discussive Logics*, in: *Paraconsistent Logic: Essays on the Inconsistent*, eds. G. Priest, R. Routley, J. Norman, Philosophia Verlag, München–Hamden–Wien 1989, pp. 227–244.

44 N.C.A. da Costa, F. Doria, *On Jaśkowski's Discussive Logics*, "Studia Logica" 1995, Vol. 54, No. 1, pp. 33–60.

45 L. de Moraes, J.M. Abe, *Some Results on Jaśkowski's Discursive Logic*, "Logic and Logical Philosophy" 2001, Vol. 9, pp. 25–33.

46 M. Urchs, *On the Role of Adjunction in Para(In)Consistent Logic*, in: *Paraconsistency: The Logical Way to the Inconsistent*, eds. W.A. Carnielli, M.E. Coniglio, I.M.L. D'Ottaviano, Marcel Dekker, New York–Basel 2002, pp. 487–499.

a conception of a paraconsistent logic as a logic where it is not true that from an inconsistency follows its conjunction.

In turn, in 2005 Janusz Ciuciura observed that da Costa and Dubikajtis's axiomatization from 1977 resulted in a different system from the original discussive logic $\mathbf{D_2}$.[47] Ciuciura introduced another axiomatization for da Costa and Dubikajtis's system – thus dealing with a discussive language with the conjunction $\wedge^{l}_{d}$ – using fewer axioms. In the next year, Ciuciura considered a variant of the discussive logic in which a discussive negation $\sim_d$ is introduced.[48] The intuitive meaning of a formula $\sim_d A$ is to be read as "some participant rejects *A*." Three years later, in 2008, Ciuciura proposed an axiomatization for $\mathbf{D_2}$ in a language with discussive connectives using $\wedge^{r}_{d}$.[49]

In 2005, Jean-Yves Béziau introduced the logic **Z** as a way to solve Jaśkowski's problem of providing a paraconsistent logic with intuitive justification, and where negation had enough properties to be consider a negation.[50] The logic is actually again motivated by discussive logic, where one "modalizes" the negation "$\Diamond\sim A$," which is accordingly understood as "it is possible that not *A*" in the modal logic **S5**.

In 2006, Joke Meheus combined a (paraconsistent) adaptive logic with discussive logic.[51] A new result came from formulating discussive logic without the discussive connectives. In another paper published in 2006, Meheus also considered an adaptive-discussive logic that permits representing discussions in which participants contradict themselves.[52]

[47] J. Ciuciura, *On the da Costa, Dubikajtis and Kotas' System of Discursive Logic*, "Logic and Logical Philosophy" 2005, Vol. 14, pp. 235–252. This seems clear given the fact that da Costa and Dubikajtis axiomatized the system with a different conjunction.

[48] J. Ciuciura, *A Quasi-Discursive System*, "Notre Dame Journal of Formal Logic" 2006, Vol. 47, No. 3, pp. 371–384.

[49] J. Ciuciura, *Frontiers of the Discursive Logic*, "Bulletin of the Section of Logic" 2008, Vol. 37, No. 2, pp. 81–92.

[50] J.-Y. Béziau, *The Paraconsistent Logic Z: A Possible Solution to Jaśkowski's Problem*, "Logic and Logical Philosophy" 2006, Vol. 15, No. 2, pp. 99–111.

[51] J. Meheus, *An Adaptive Logic Based on Jaśkowski's Approach to Paraconsistency*, "Journal of Philosophical Logic" 2006, Vol. 35, No. 6, pp. 539–567.

[52] J. Meheus, *Discussive Adaptive Logics: Handling Internal and External Inconsistencies*, in: *Essays in Logic and Ontology*, eds. J. Malinowski, A. Pietruszczak, Rodopi, Amsterdam–New York 2007, pp. 211–223.

In 2007, D'Ottaviano and Carlos Hifume proposed a paraconsistent modal logic.[53] This logic can be seen as a kind of Jaśkowski's discussive logic that could be used, in general, as a deductive logic of science. In 2015, Ciuciura contributed to the study of the algebra of the discussive logic $\mathbf{D_2}$,[54] basing his study on the work of Jerzy Kotas.

Besides the previous works, the contemporary investigation on discussive logic has been mainly carried out in Toruń by Andrzej Pietruszczak, Marek Nasieniewski, and Krystyna Mruczek-Nasieniewska. Below, I present a succinct annotated bibliography of their work on discussive logic:

2001 Nasieniewski, *A Comparison of Two Approaches to Paraconsistency: Flemish and Polish*. In this work, Marek Nasieniewski compared the approaches to paraconsistent logic of discussive logic and of adaptive logics.[55]

2005 Mruczek-Nasieniewska and Nasieniewski, *Syntactical and Semantical Characterization of a Class of Paraconsistent Logics*. In this work, Mruczek-Nasieniewska and Nasieniewski built upon Béziau's 2005 article to present different paraconsistent logics using normal modal logics other than **S5**.[56] In the same year, similar observations were made by João Marcos.[57]

2008 Mruczek-Nasieniewska and Nasieniewski, *Paraconsistent Logics Obtained by J.-Y. Béziau's Method by Means of Some Non-Normal Modal Logics*. In this work, Mruczek-Nasieniewska and Nasieniewski generalized their work from 2005 to present more paraconsistent logics using regular modal logics.[58]

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53 I.M.L. D'Ottaviano, C. Hifume, *Peircean Pragmatic Truth and da Costa's Quasi-Truth*, "Studies in Computational Intelligence (SCI)" 2007, Vol. 64, pp. 383–398.

54 J. Ciuciura, *Algebraization of Jaśkowski's Paraconsistent Logic D2*, "Studies in Logic, Grammar and Rhetoric" 2015, Vol. 42, No. 1, pp. 173–193.

55 M. Nasieniewski, *A Comparison of Two Approaches to Paraconsistency: Flemish and Polish*, "Logic and Logical Philosophy" 2001, Vol. 9, pp. 47–74.

56 K. Mruczek-Nasieniewska, M. Nasieniewski, *Syntactical and Semantical Characterization of a Class of Paraconsistent Logics*, "Bulletin of the Section of Logic" 2005, Vol. 34, No. 4, pp. 229–248.

57 J. Marcos, *Nearly Every Normal Modal Logic Is Paranormal*, "Logique & Analyse" 2005, Vol. 48, No. 189/192, pp. 279–300.

58 K. Mruczek-Nasieniewska, M. Nasieniewski, *Paraconsistent Logics Obtained by J.-Y. Béziau's Method by Means of Some Non-Normal Modal Logics*, "Bulletin of the Section of Logic" 2008, Vol. 37, pp. 185–196.

2008 Nasieniewski and Pietruszczak, *The Weakest Regular Modal Logic Defining Jaśkowski's Logic D_2*. In this work, Nasieniewski and Pietruszczak considered the smallest regular modal logic which enables defining the discussive logic $\mathbf{D_2}$.[59]

2009 Mruczek-Nasieniewska and Nasieniewski, *Béziau's Logics Obtained by Means of Quasi-Regular Logics*. In this work, Mruczek-Nasieniewska and Nasieniewski generalized their work from 2005 and 2008 to present more paraconsistent logics using quasi-regular modal logics.[60]

2011 Nasieniewski and Pietruszczak, *A Method of Generating Modal Logics Defining Jaśkowski's Discussive Logic D_2*. In this paper, Nasieniewski and Pietruszczak provided a method of obtaining various modal logics that can be used to define the discussive logic $\mathbf{D_2}$ via the translation tr(x).[61]

2012 Nasieniewski and Pietruszczak, *On the Weakest Modal Logics Defining Jaśkowski's Logic D_2 and the D_2-Consequence*. In this work, Nasieniewski and Pietruszczak indicated the weakest modal logic that can be used to define the discussive logic $\mathbf{D_2}$.[62] They specified that the discussive logic $\mathbf{D_2}$ can be presented either as a set of discussive formulas or as a consequence relation, and they provided the weakest modal logic for any of the two presentations of the discussive logic.

2013 Nasieniewski and Pietruszczak, *On Modal Logics Defining Jaśkowski's D_2-Consequence*. In this work, Nasieniewski and Pietruszczak studied normal and regular modal logics that can be used to define the discussive logic $\mathbf{D_{2}}$-consequence.[63]

2014 Nasieniewski and Pietruszczak, *Axiomatization of Minimal Modal Logics Defining Jaśkowski's-Like Discussive Logics*. In this work, Nasieniewski

[59] M. Nasieniewski, A. Pietruszczak, *The Weakest Regular Modal Logic Defining Jaśkowski's Logic D_2*, "Bulletin of the Section of Logic" 2008, Vol. 37, pp. 197–210.

[60] K. Mruczek-Nasieniewska, M. Nasieniewski, *Béziau's Logics Obtained by Means of Quasi-Regular Logics*, "Bulletin of the Section of Logic" 2009, Vol. 38, pp. 189–203.

[61] M. Nasieniewski, A. Pietruszczak, *A Method of Generating Modal Logics Defining Jaśkowski's Discussive Logic D_2*, "Studia Logica" 2011, Vol. 97, pp. 161–182.

[62] M. Nasieniewski, A. Pietruszczak, *On the Weakest Modal Logics Defining Jaśkowski's Logic D_2 and the D_2-Consequence*, "Bulletin of the Section of Logic" 2012, Vol. 41, pp. 215–232.

[63] M. Nasieniewski, A. Pietruszczak, *On Modal Logics Defining Jaśkowski's D_2-Consequence*, in: *Paraconsistency: Logic and Applications*, eds. K. Tanaka, F. Berto, E. Mares, F. Paoli, Springer, Dordrecht 2013, pp. 141–160.

and Pietruszczak presented axiomatizations of the minimal modal logic that can be used to define some variants of discussive logics.[64]

2017 Mruczek-Nasieniewska and Nasieniewski, *Logics with Impossibility as the Negation and Regular Extensions of the Deontic Logic D_2*. In this work, Mruczek-Nasieniewska and Nasieniewski built upon Béziau's work from 2005 and considered negation to be defined as impossibility, instead of unnecessity. This helped obtain expressibility of analogous logics to logic **Z** using regular logics being extension of the smallest regular deontic logic.[65]

2018 Mruczek-Nasieniewska and Nasieniewski, *A Characterization of Some Z-Like Logics*. The logic **Z** was characterized similarly as the discussive logic $\mathbf{D_2}$ by using the modal logic **S5**.[66] By considering other modal systems than **S5** one can define different **Z**-like logics. In Mruczek-Nasieniewska and Nasieniewski's work, different **Z**-like logics are studied by considering other modal logics. The authors took two negations to be understood as unnecessity and as impossibility, respectively.

2019 Mruczek-Nasieniewska, Nasieniewski and Pietruszczak, *A Modal Extension of Jaśkowski's Discussive Logic D_2*. In this work, Mruczek-Nasieniewska, Nasieniewski and Pietruszczak considered a version of discussive logic where participants can express the modal status of their assertions.[67] In the discussive logic $\mathbf{D_2}$, an assertion preceded by the symbol $\Diamond$ can only express that a given statement is possible from the point of view of an observer of the discussion. The idea is thus to allow participants of the discussion to use $\Diamond A$ to express statements of the kind "it is possible that *A*."

2019 Mruczek-Nasieniewska and Nasieniewski, *A Kotas-Style Characterization of Minimal Discussive Logic*. In this work, Mruczek-Nasieniewska

64 M. Nasieniewski, A. Pietruszczak, *Axiomatization of Minimal Modal Logics Defining Jaśkowski's-Like Discussive Logics*, in: *Trends in Logic XIII: Gentzen's and Jaśkowski's Heritage. 80 Years of Natural Deduction and Sequent Calculi*, eds. A. Indrzejczak, J. Kaczmarek, M. Zawidzki, Wydawnictwo Uniwersytetu Łódzkiego, Łódz 2014, pp. 149–163.

65 K. Mruczek-Nasieniewska, M. Nasieniewski, *Logics with Impossibility as the Negation and Regular Extensions of the Deontic Logic D_2*, "Bulletin of the Section of Logic" 2017, Vol. 46, pp. 261–280.

66 K. Mruczek-Nasieniewska, M. Nasieniewski, *A Characterization of Some Z-Like Logics*, "Logica Universalis" 2018, Vol. 12, https://doi.org/10.1007/s11787-018-0184-9.

67 K. Mruczek-Nasieniewska, M. Nasieniewski, A. Pietruszczak, *A Modal Extension of Jaśkowski's Discussive Logic D_2*, op. cit., pp. 451–477.

and Nasieniewski considered syntactical characterization of a minimal variant of the discussive logic $\mathbf{D}_2$.[68] Instead of considering that each participant has access to the assertions of all other participants of the discussion, they explored the idea that a participant must have access to the assertions of at least one participant in the discussion.

2020 Mruczek-Nasieniewska and Nasieniewski, *On Correspondence of Standard Modalities and Negative Ones on the Basis of Regular and Quasi-Regular Logics*. In this work, Mruczek-Nasieniewska and Nasieniewski investigated different **Z**-like logics considering negation to be defined as unnecessity.[69]

Finally, it is worth mentioning that recently, in 2018, Hitoshi Omori and Jesse Alama showed that Ciuciura's axiomatization from 2006 and 2008 of the discussive logic and its variant required some corrections, and presented the final axiomatization for it.[70] Omori is also trying to find connections between different paraconsistent logics, particularly the discussive logic $\mathbf{D}_2$ and Florencio González-Asenjo's/Graham Priest's logic **LP**.[71] Edelcio G. de Souza, Alexandre Costa-Leite and Diogo H.B. Dias, in turn, in a new approach to paraconsistency called "paraconsistentization" – aimed at studying how a given logic can be transformed into a paraconsistent logic – observed that the discussive logic $\mathbf{D}_2$ can be considered a paraconsistent logic that results from adding discussive operators.

5. The Discussive Tradition

In this section, I offer some reflections on the tradition started by Jaśkowski and his collaborators, and continued by many scholars.

[68] K. Mruczek-Nasieniewska, M. Nasieniewski, *A Kotas-Style Characterization of Minimal Discussive Logic*, "Axioms" 2019, Vol. 8, No. 4, https://doi.org/10.3390/axioms8040108.

[69] K. Mruczek-Nasieniewska, M. Nasieniewski, *On Correspondence of Standard Modalities and Negative Ones on the Basis of Regular and Quasi-Regular Logics*, "Studia Logica" 2020, Vol. 108, pp. 1087–1123.

[70] H. Omori, J. Alama, *Axiomatizing Jaśkowski's Discussive Logic* $\boldsymbol{D}_2$, "Studia Logica" 2018, Vol. 106, No. 6, pp. 1163–1180.

[71] See H. Omori, *Observations on Jaśkowski's Discussive Logic*, in: *Proceedings of XI Conference "Smirnov Readings in Logic"*, ROIFN, Moscow 2019, pp. 77–79.

As can be seen from the history of discussive logic, the unity and identity of the Polish approach to paraconsistency is to a great extent determined by the previous tradition of the Lvov-Warsaw School and international collaboration. The creation of discussive logic constituted an effort to express a contradiction in a way that does not trivialize a system. Given a consistent system (that is, classically, a non-trivial system), a given contradiction would not overfill it with any sentence, since some contradictions are being translated into consistent expressions of this consistent system. Moreover, Jaśkowski's interest in inconsistency was linked to Łukasiewicz's investigation on the principle of contradiction. By a critical survey on defences of the principle of non-contradiction, Łukasiewicz arrived at the conclusion that that principle had not been well motivated from a philosophical point of view. As a mathematician, Jaśkowski knew how to take advantage of modal logic to represent a possibility where a contradiction could be consistently true.

Furthermore, the discussive tradition was expanded by a common interest by da Costa and his collaborators. Had it not been for the joint work of da Costa and Jaśkowski's students, Jerzy Kotas and Lech Dubikajtis, the tradition probably would not have the fame that it enjoys today.

By sketching the development of discussive logic, I tried to present a model for the Polish approach to paraconsistency initiated by Jaśkowski in the late 1940s. This was continued by Kotas and Dubikajtis from the 1960s to the 1980s – with input from da Costa, Achtelik, Dudek, Konior, Furmanowski, Dziobiak, Błaszczuk, Urchs, Doria, and Pieczkowski – and reinstated by Perzanowski in the 1990s. Since then, the contributions on discussive logic have been led by Pietruszczak, Nasieniewski, and Mruczek-Nasieniewska in Toruń.

The investigation on discussive logic also brought new investigations on philosophical logic. As I have tried to remark in the text, the international collaboration of Polish and Brazilian logicians, and the subsequent effort to join different systems of logic, including paraconsistent ones, led to expansions of discussive logic to set theory, new systems that take into account the representation of causal relations, and the idea that several connectives can be taken to be modal.

If there is a moral to be drawn from this historical tradition, one can consider the following: it is a *sine qua* condition for the development of the humanities and science to work on the ideas of a given school's master, and to try to participate actively in international collaborations by taking interest in the current research of our colleagues. By following this formulation, one can secure the future development of any intellectual tradition, and the survival of long-term schools of thought.

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