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A Brazilian Southern School in the Philosophy of Physics: Identity, Indistinguishability, and Individuality in Quantum Theories

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Abstract: This paper presents the works developed by a group of researchers mainly in the South of Brazil on the philosophy of physics, dealing mostly with the logic and metaphysics of the notions of identity, indistinguishability, and individuality of quantum entities.

Key words: South of Brazil, philosophy of physics, identity and individuality of quantum entities, non-individuality, quantum metaphysics, quasi-set theory

> The sameness of a particle is not an absolute concept. It has only a restricted significance and breaks down completely in some circumstances.
>
> Erwin Schrödinger, *What Is an Elementary Particle?*

1. Introduction

The origins of the works to be covered here go back to the motivations provided by Newton da Costa, and the works by Steven French on the philosophy of quantum mechanics. In his book *Ensaio sobre os fundamentos da lógica* [Essay on the Foundations of Logic], published in 1980,[1] da Costa argued that any logical principle can be questioned ("dialectized," as he preferred to say following Gaston Bachelard), in particular, this can be done also with respect to the basic notions of classical logic, and he specifically discusses the Principle of Identity in

[1] N.C.A. da Costa, *Ensaio sobre os fundamentos da lógica*, Hucitec, São Paulo 1980.

the first-order formulation, namely, "For all x, $x = x$." By questioning a certain principle P, he means to construct a "reasonable" logical system where P does not hold in general.[2] By "reasonable" he understands a logical system endowed with clear syntactic and well-established semantics.

Steven French, since his PhD thesis at the University of London in the 1980s, has presented important work on the philosophical foundations of quantum mechanics, mainly by considering the individuality of quantum entities.[3] In the late 1980s, he was working at the State University of Campinas and I had the opportunity to contact him just before finishing my own dissertation. His works on the validity of the Principle of the Identity of the Indiscernibles (one-half of Leibniz's Law, see below) in quantum mechanics have become key references in the field.[4]

Da Costa was occupied with logic. In order to inspire a possible departure from the Principle of Identity, he found in Erwin Schrödinger's ideas a motivation for the elaboration of a system where such a principle does not hold in full. Schrödinger said, in his book *Science and Humanism*[5] and in his essay *What Is an Elementary Particle?*,[6] that the notion of identity (sameness) does not make sense for the elementary particles in quantum physics. The reason, we can say today, is that we cannot discern among the particles of the same kind when joined in a collection, and also that when they are described by an entangled state, we cannot identify them in a "which is which" way, contradicting the standard notion of identity of classical logic, where individuals can carry names that act as rigid designators, naming the same object in all possible worlds or contexts. Schrödinger does not enter into logical discussions, and also did not mention the theory of identity of standard logic, but poses the challenge concerning the application of this notion to the fundamental entities dealt with by quantum physics. It seems clear that the notion of identity Schrödinger is looking for is linked to the physi-

[2] Ibid., p. 124.

[3] S. French, *Identity and Individuality in Quantum Theory*, in: *The Stanford Encyclopedia of Philosophy* (Winter 2019 Edition), ed. E.N. Zalta, URL: https://plato.stanford.edu/archives/win2019/entries/qt-idind/ (substantive revision on 30.10.2019).

[4] S. French, M. Redhead, *Quantum Physics and the Identity of Indiscernibles*, "British Journal for the Philosophy of Science" 1988, Vol. 39, pp. 233–246; S. French, *Identity and Individuality in Classical and Quantum Physics*, "Australasian Journal of Philosophy" 1989, Vol. 67, pp. 432–446.

[5] E. Schrödinger, *Nature and the Greeks and Science and Humanism*, Cambridge University Press, Cambridge 2004.

[6] E. Schrödinger, *What Is an Elementary Particle?*, in: E. Schrödinger, *Science, Theory and Man*, George Allen and Unwin, London 1967, pp. 193–223.

cal notion of re-interpretation: a thing endowed with identity must be recognized as such in other circumstances, or contexts, and, clearly, this is not what happens with quantum entities.

This intuitive notion of identity can be associated with the (also informal) notion of an *individual*: an individual would be something that (1) is a *one* of a kind, or of a certain type, say a person, a chair, a pen, and (2) can be re-identified as such, that is, as being *that* individual in different contexts. This is supposed to hold with persons, chairs and pens, although we should take care with Hume's remarks that such confidence in the permanence of the object's identity is only a fiction of our imagination.[7] It seems quite clear that quantum objects seem not to be individuals in this sense, although some such as the Bohmians could contest that quantum objects obey such conditions.[8]

As said before, da Costa was occupied with logical considerations.[9] By believing that the Principle of Identity can be questioned he requested the existence of a reasonable logical system where this principle does not hold in full. Thus, in order to sustain such a thesis, he sketched a first-order two-sorted logic he called "Schrödinger Logic" with the following characteristics. Beyond the standard logical symbology of first-order systems with identity, he assumed two kinds of individual variables and also corresponding individual constants, denoted by x, x', x'', … and X, X', X'', … for the individual variables of the first and of the second species respectively. The novelty is that only expressions of the form $t = u$ are formulas if and only if both t and u are terms (individual variables or individual constants) of the second kind. So, the language does not make reference to the identity (or to the difference) of objects denoted by the terms of the first species. Consequently, the Principle of Identity in the form does not hold in full. If the terms of the first species are designating elementary particles, then Schrödinger's idea gets vindicated, since the language of the given logic (suitable postulates are provided) does not speak of their identity or of their differences.

7 D. Hume, *Treatise of Human Nature*, ed. L.A. Selby-Bigge, Oxford Univeristy Press, Oxford 1985, pp. 200–201.

8 In Bohmian quantum mechanics, particles have trajectories and the trajectories serve to provide particles' identities. But it should be remarked that the positions are ascribed by *hidden* variables and cannot be known. It seems to me that this is a mystery even greater than to suppose that quantum objects simply do not conform to the given definition of an individual.

9 N.C.A. da Costa, *Ensaio sobre os fundamentos da lógica*, op. cit., pp. 117ff.

From the syntactical point of view, the logic works. But problems arise with its semantic counterpart. Da Costa suggests that a semantics could be developed by taking two non-empty sets D_1 and D_2, being $D_2 \subset D_1$, so that the individual constants of the first species are in correspondence with D_1. The n-ary predicates of the language are associated with relations in D_1^n. To the individual constants of the second species, we associate elements of D_2. Thus he emends that "[n]aturally, to the symbol of equality one associates the equality relation over D_2."[10] So, he suggests, all the semantic results related to the logic can be obtained without difficulty, so that one can get both the soundness of this semantics and the completeness of the system relative to such a semantics.

But, da Costa reminds us, such a semantics brings philosophical difficulties. Really, according to him, D_1 should not be taken as a *set* strictly speaking, since the relation of identity would lack sense in general; only for the elements of D_2 such a relation can be stated so that they are equal or distinct. Thus, he says that "in order to surpass this difficulty, there are two open roads: 1. to try to generalize the notion of set, for instance by building a theory of *quasi-sets* containing the standard sets as special cases, and to found a semantics for the system in such a theory; 2. one will not try to build a formal semantics for the system, but an *informal semantics*, with the help of the natural language, a little bit imprecise but taking into account what quantum mechanics says."[11] This second alternative finds its reason in da Costa's belief that "in the basis of all deductive sciences there is an informal semantics."[12]

As a motivation for his PhD thesis under da Costa's supervision, this paper's author has taken the above challenges into consideration.

2. The Thesis

My thesis was titled *Não-Reflexividade, Indistinguibilidade e Agregados de Weyl* [Non-Reflexivity, Indistinguishability, and Weyl's Aggregates];[13] the logics that depart from the standard theory of identity of classical logic, in particular violat-

[10] Ibid., p. 119.
[11] Ibid.
[12] Ibid., p. 120.
[13] D. Krause, *Não-Reflexividade, Indistinguibilidade e Agregados de Weyl*, PhD dissertation, University of São Paulo, 1990.

ing the Principle of Identity, as da Costa's Schrödinger Logic, were termed *non-reflexive*, once this principle is also known as the "reflexive law of identity." In one of the chapters, da Costa's first-order system was extended to a higher-order logic of order omega (simple theory of types) and a Henkin semantics was proposed, with a weak completeness theorem proven. We remark that, as in the case delineated by da Costa for his system, such semantics was elaborated in a standard set theory (you can think of the ZFC system).

The reasons to develop such a higher-order system were, first, to get a generalization of da Costa's system but, second, and perhaps mainly, in a higher-order language, we can formulate Leibniz's Law in full, hence *defining* identity, and, in particular, we can consider the standard formulations of Leibniz's Principle of the Identity of Indiscernibles, a subject that at that time was being considered in the foundations of quantum theories; philosophers were disputing its validity in such a field.[14] Leibniz's Law can be written

$$x = y := \forall F\,(Fx \leftrightarrow Fy) \qquad (1)$$

where x and y are terms of type τ and F is a variable of type $\langle\tau\rangle$. The sufficient condition is the Principle of the Identity of Indiscernibles, while the necessary one is the Principle of the Indiscernibility of Identicals.[15] In other words, by Leibniz's Law, identity is introduced *via* indiscernibility (agreement with respect to *all* predicates). So, if we can *define* identity for all objects, how to maintain the idea that it does not hold for some of them? Thus, the relation of indiscernibility (or "indistinguishability") was used instead of identity in Leibniz's Law, meaning that entities that share all their characteristics are indiscernible, and not identical. Thus, being "≡" a binary predicate symbol, x and y terms of type τ and being F a variable of type $\langle\tau\rangle$, we put

$$\mathrm{x} \equiv \mathrm{y} := \forall F\,(Fx \leftrightarrow Fy) \qquad (2)$$

to mean that and are indiscernible. But this would be just a change of terminology since the definition would be the same as standard Leibniz's Law. Anyway, the interesting fact is that using higher-order languages we are able to express things such as the "definition" of identity of elementary particles as given by J.M. Jauch,

[14] S. French, *Identity and Individuality*, op. cit.; S. French, *Why the Principle of the Identity of Indiscernibles Is Not Contingently True Either*, "Synthese" 1989, Vol. 48, pp. 141–166; S. French, M. Redhead, *Quantum Physics and the Identity of Indiscernibles*, op. cit.

[15] Frequently the Principle of the Indiscernibility of Identicals is what is called "Leibniz's Law."

namely, “two elementary particles are identical if (and only if) they agree in all their intrinsic properties.”[16] Intrinsic properties are those properties that do not depend on space and time, such as electric charge, mass or spin. Thus, if P is a variable whose arguments are properties of individuals (hence we are going now to a third-order logic), we can define

$$x \equiv_P y := \forall F\,(P(F) \to (Fx \leftrightarrow Fy)) \qquad (3)$$

that is, x and y are P-indiscernible if they agree with respect to every property that satisfies P, which is a variable of type $\langle\langle\tau\rangle\rangle$. If P stands for “intrinsic property,” we arrive at Jauch’s definition with much more precision. It should be remarked that Jauch’s definition confuses the notions of identity (agreement with respect to *all* properties) with indiscernibility relative to intrinsic properties only.

But despite this more expressive language, the challenge remains: how to differentiate between identity (“=”), given by Leibniz’s Law (1), and indiscernibility (2)? Classical logic defines the first in terms of the second, so first of all we need to break this correlation. One of the options is to go to more than the object’s properties and relations. *Haecceity* was a term coined in the Middle Ages to mean some characteristics that make the individual the individual it is so that it can be referred to as “this one.” Thus, by admitting the existence of haecceities of some kind, we can go beyond the properties and qualities of things and suppose that there is something more than properties and relations to give them their identities, and so Leibniz’s Law can be violated. But this seems to be a radical move: if haecceities (by definition) can be reduced to neither properties nor relations, how can we deal with them? Anyway, this is a supposition that appears in classical physics. When we say that classical particles of the same kind obey Maxwell–Boltzmann statistics, we are agreeing that they have all the same properties but that even so a permutation of them conduces to a different state; hence, *something more* is being presupposed, what Heinz Post called *transcendental individuality*.[17] But this is not so in the quantum realm. Quantum entities do not obey Maxwell–Boltzmann statistics, but either Bose–Einstein or Fermi–Dirac, and in both cases their indistinguishability is assumed (as suggested by Post), *right from the start*. Post’s claim was important here; the search for a mathematics where indiscernibility was not *made by hand*, for example, when one assumes

[16] J.M. Jauch, *Foundations of Quantum Mechanics*, Addison-Wesley, Boston, MA, 1968, p. 275.
[17] H. Post, *Individuality in Physics*, “Vedanta for East and West” 1973, Vol. 132, pp. 14–22.

symmetry postulates (an analogy would be to confine the entities to deformable, not rigid structures having more automorphisms than the identity function), but where the notion of indiscernibility was primitive.

The second alternative would be to find some suitable semantics for Schrödinger Logic, and here a theory of *quasi-sets* enters the scene. In fact, if we can admit the existence of "sets" (really, *quasi-sets*) such that an indistinguishability relation can hold for all objects but identity does not, we can have indiscernible but not identical entities, thus giving life to both equations (1) and (2) without conflating identity. So, a theory of quasi-sets is in need, where a distinction between indistinguishability and identity is given *without* assuming haecceities.

The first attempt to develop such a theory was another chapter of my thesis. The result was published later[18] and improvements in the theory continue to this day, thirty years after the first steps. This trajectory shows the difficulty there is in trying to suspend the universal application of identity. Some philosophers repute this notion as a fundamental one,[19] while others contest such an assumption.[20] We can repute this notion not as a necessary one, except perhaps in standard mathematics, as I shall comment on at the end, but it is quite useful and simplifies the discourse also in the empirical sciences. Below I shall provide some hints about the theory of quasi-sets.

Weyl's aggregates (mentioned in the title of my thesis) entered the work for the following reason. In his masterpiece,[21] Hermann Weyl discussed in Appendix B the aggregation of individuals. His aim was to explain how things such as elementary particles are considered in quantum mechanics. Weyl says that "that what imports" in quantum mechanics is not the identity of the particles, but the "ordered decompositions" like (4) below, which expresses, given n particles of

[18] D. Krause, *On a Quasi-Set Theory*, "Notre Dame Journal of Formal Logic" 1992, Vol. 33, No. 3, pp. 402–411.

[19] O. Bueno, *Why Identity Is Fundamental*, "American Philosophical Quarterly" 2014, Vol. 51, No. 4, pp. 325–332.

[20] D. Krause, J.R.B. Arenhart, *Is Identity Really So Fundamental?*, "Foundations of Science" 2019, Vol. 24, No. 1, pp. 51–71; D. Krause, J.R.B. Arenhart, *Does Identity Hold A Priori in Standard Quantum Mechanics?*, in: *Probing the Meaning and Structure of Quantum Mechanics: Entanglement, Relations and Information*, eds. D. Aerts, M.L. Dalla Chiara, C. de Ronde, D. Krause, World Scientific, Hackensack, NJ–London 2019, pp. 99–120.

[21] H. Weyl, *Philosophy of Mathematics and Natural Science*, Princeton University Press, Princeton, NJ, 1949.

the same species (hence indistinguishable), how many of them there are in each particular state being considered; so, we shall have things like

$$n = n_1 + n_2 + \cdots + n_k \qquad (4)$$

which says that we have n_i particles in the state C_i. In order to express that, he takes the set S of the n particles (notice that S is assumed to be a *set*, and this will be relevant soon) and an equivalence relation "$\sim$" over this set. Then the C_i can be seen as the equivalence classes and, by considering their cardinalities, we get (4).

The challenge, not considered by Weyl, is that, as said before, S is a set, hence even if we take the C_i to represent the states of the particles, the standard theory of identity (STI) applies to them and so we cannot have *just* the ordered decomposition (4) without being committed to the identity of the elements of the equivalence classes. STI is included in standard logic and mathematics and says that given any two objects, they are different and then (due to Leibniz's Law) do present a difference. In such frameworks, *there are no indiscernible but not identical objects*. There is no escape; within a standard set theory, every represented entity becomes an individual and the most we can do is to *simulate* indiscernibility, but not consider it as it should be taken, as something holding *right from the start*, as it seems to be the case with indistinguishable quantum entities.

Another important link with the subjects of the thesis was made with M.L. Dalla Chiara and G. Toraldo di Francia's work. They were working basically on the same subject.[22] They developed a theory of *quasets* in order to cope with collections of quantum objects, also questioning the applicability of standard set theories to deal with collections of indiscernible quantum entities. A comparison between their theory of quasets and the theory of quasi-sets was done in the paper *Quasi Set Theories for Microobjects: A Comparison*,[23] and an extension of their theory relating it to *rough sets* is done in *Un acercamiento a las semánticas Nmatriciales basadas en QST*.[24] Basically, in the theory of quasets, identity holds for all objects, but the membership relation is weakened so that if we have

22 See, e.g., M.L. Dalla Chiara, G. Toraldo di Francia, *Individuals, Kinds and Names in Physics*, in: *Bridging the Gap: Philosophy, Mathematics, and Physics*, eds. G. Corsi, M.L. Dalla Chiara, G.C. Ghirardi, Kluwer Academic Publishers, Dordrecht 1993 [1978], pp. 261–284.

23 M.L. Dalla Chiara, R. Giuntini, D. Krause, *Quasi Set Theories for Microobjects: A Comparison*, in: *Interpreting Bodies: Classical and Quantum Objects in Modern Physics*, ed. E. Castellani, Princeton University Press, Princeton, NJ, 1998, pp. 142–152.

24 J.P. Jorge, F. Holik, D. Krause, *Un acercamiento a las semánticas Nmatriciales basadas en QST*, forthcoming.

a quaset A and an object a, we can say that "a certainly belongs to A" by writing "$a \in A$," that "a certainly does not belong to A" by "$a \notin A$," but is not equivalent to this last formula; thus we have as a theorem that $a \notin A \to \neg(a \in A)$ but not the other way around. So, if a belongs to A, we can conclude that it is false that it certainly does not belong to A, and so we get something like a fuzzification of the situation.

3. Quasi-Sets and Applications

The idea of quasi-sets is different from that of quasets. In quasi-set theory, identity does not hold for all objects. So, we may have quasi-sets whose elements are completely indiscernible from each other; membership works as usual, but identity does not. The core of the theory is ZFA, the Zermelo–Fraenkel set theory with atoms. But the theory admits the existence of another kind of atoms, for which the standard notion of identity does not hold. So, we have M-atoms, which behave as the atoms in ZFA, and the m-atoms, to which identity does not apply; this is achieved by saying that expressions like "$x = y$" are not well-formed if x or y denotes an m-atom. But the primitive relation of indiscernibility, symbolized by "$\equiv$" holds for all objects; of course $x = y \to x \equiv y$, but not conversely. The axioms provide the way to construct a universe of quasi-sets, which turns to be a non-rigid (deformable) structure.

A quasi-set may have a cardinal, termed its *quasi-cardinal*, given axiomatically since quasi-cardinals are not defined by means of ordinals as usual (or relying on the notion of ordinal), despite having properties like those of standard cardinals; so, we may have certain quantities of entities that cannot be ordered, counted, labelled "significantly" (that is, so that a proper name does not act as a rigid designator). But, for the purpose of this article, the most important thing is not to speak about the details of the theory, but of its applications done by the "southern group." It is important to notice that some philosophers claim that once a collection has a cardinal greater than one, its elements are necessarily distinct. This can be assumed within a standard set theory, where a set is just a collection of *distinct* entities, but not in the theory of quasi-sets.[25]

[25] For a discussion on this topic, see D. Krause, *On Identity, Indiscernibility, and Individuality in the Quantum Domain*, forthcoming.

In the 1990s, Adonai S. Sant'Anna, from the Department of Mathematics of the Federal University of Paraná, who had obtained a master's degree in physics from the same university, started working on quasi-sets and applications to quantum mechanics. He received his PhD also with Newton da Costa at the University of São Paulo, and he wrote a series of papers, one of them with Analice G. Volkov, who was also studying with da Costa and became a member of the same department (Analice died in a bus crash in 2001 before finishing her PhD). In these papers, working in the mathematical framework provided by the theory of quasi-sets, the authors were able to assume the indiscernibility of the quantum entities as a primitive notion and not as something obtained a posteriori by making some trick with identity (as in Weyl's case). The "quantum statistics" arose naturally from the formalism, and the Indistinguishability Postulate, an essential assumption in standard quantum mechanics, was no more assumed as a postulate, for it results from the hypothesis of the indiscernibility, as is intuitive that it should be so.[26]

In 2000, I moved to the Department of Philosophy of the Federal University of Santa Catarina (UFSC), in another state of the South, whose capital is Florianópolis. There, he started working in logic with some students and a colleague, Antonio M.N. Coelho, a member of the Department of Philosophy. Antonio, who has quite a good background in logic and in mathematics, and has obtained his PhD also with da Costa in São Paulo. Together we worked on mathematical structures and wrote a paper on the subject[27] and supervised some graduate works on the subject; in *Observa*ções *sobre a neutralidade ontológica da matem*ática,[28] the authors argue that standard set theories are not "neutral" ontologically, as some suppose, since they cannot represent adequately an ontology of non-individuals. Later, a former student of that university, Jonas R.B. Arenhart, finished his PhD at UFSC working with quasi-sets and joined the group. After a period in another university, he entered the Department of Philosophy of UFSC. With Jonas, a series of papers dealing more with the philosophical aspects of indiscernibility

[26] The interested reader can consult A.S. Sant'Anna, D. Krause, *Hidden Variables and Indistinguishable Particles*, "Foundations of Physics Letters" 1997, Vol. 10, pp. 409–426; D. Krause, A.S. Sant'Anna, A.G. Volkov, *Quasi Set Theory for Bosons and Fermions*, "Foundations of Physics Letters" 1999, Vol. 12, No. 1, pp. 67–79.

[27] D. Krause, A.M.N. Coelho, *Identity, Indiscernibility, and Philosophical Claims*, "Axiomathes" 2005, Vol. 15, No. 2, pp. 191–210.

[28] G. Gelowate, D. Krause, A.M.N. Coelho, *Observações sobre a neutralidade ontológica da matemática*, "Episteme" 2005, Vol. 17, pp. 145–157.

has appeared, a work that continues today (see the references). He also has quite a good list of publications involving several aspects of the metaphysics of quantum theories. Later other students became interested in the subject and today we can mention Raoni W. Arroyo, who obtained his PhD also from UFSC and started working on the metaphysics of quantum theories.[29] These authors have pointed to important details involving the metaphysics of science, and advanced the idea that there may exist several different and alternative images of the world provided by a particular theory; these ideas are quite similar to those given in my 2019 work on perspectivism, whose motivation was provided by José Ortega y Gasset's notions of *perspectivism*.

In 2000, I created the research group Lógica e Fundamentos da Ciência (Logic and Foundations of Science) linked to the Diretório dos Grupos de Pesquisa of the CNPq, the Brazilian Council for Scientific Development; the group congregates several researchers and students and can be accessed online.[30] Another important contributor (and member of the group) is Otávio Bueno, who obtained his PhD in Leeds with S. French and is today at the University of Miami; the works of the group continued by emphasizing the *metaphysics of non-individuality*[31] and the fundamentality of the notion of identity.[32]

29 J.R.B. Arenhart, R.W. Arroyo, *Floating Free from Physics: The Metaphysics of Quantum Mechanics*, "arXiv:2012.05822," https://doi.org/10.48550/arXiv.2012.05822; J.R.B. Arenhart, R.W. Arroyo, *Back to the Question of Ontology (and Metaphysics)*, "Manuscrito" 2021, Vol. 44, No. 2, pp. 1–51; J.R.B. Arenhart, R.W. Arroyo, *On Physics, Metaphysics, and Metametaphysics*, "Metaphilosophy" 2021, Vol. 52, No. 2, pp. 1–25; J.R.B. Arenhart, R.W. Arroyo, *The Epistemic Value of Metaphysics*, "Synthese" 2022, Vol. 200, No. 4, https://doi.org/10.1007/s11229-022-03833-5; R.W. Arroyo, J.R.B. Arenhart, *A (meta)metafísica da ciência: o caso da mecânica quântica não relativista*, "Kriterion" 2022, Vol. 152, pp. 275–296; R.W. Arroyo, J.R.B. Arenhart, *The Powers of Quantum Mechanics: A Metametaphysical Discussion of the "Logos Approach"*, "Foundations of Science" 2022, https://doi.org/10.1007/s10699-022-09837-1.

30 *Research Group in Logic and Foundations of Science (CNPq)*, URL: https://sites.google.com/view/logicandfoundationsofscience/home?authuser=0.

31 D. Krause, J.R.B. Arenhart, O. Bueno, *The Non-Individuals Interpretation of Quantum Mechanics*, in: *The Oxford Handbook of the History of Quantum Interpretations*, eds. O. Freire Jr., G. Bacciagaluppi, O. Darrigol, T. Hartz, C. Joas, A. Kojevnikov, O. Pessoa Jr., Oxford University Press, Oxford 2022, pp. 1135–1154.

32 O. Bueno, *Why Identity Is Fundamental*, op. cit.; D. Krause, J.R.B. Arenhart, *Is Identity Really So Fundamental?*, op. cit.; D. Krause, J.R.B. Arenhart, *Does Identity Hold A Priori in Standard Quantum Mechanics?*, op. cit.

4. The Group of Florianópolis

Newton da Costa retired from the University of São Paulo in 2000. In 2003, he moved to Florianópolis and was incorporated into the graduate course in philosophy at UFSC, the Federal University of Santa Catarina. Our seminars gained much with his participation and other students were formed and become professors in different places. Other UFSC students should be mentioned: Kherian Gracher, a logician, who is presently in a post-doc researcher at the Federal University of Rio de Janeiro, and Jaison Schinaider, who started working on the notions of indistinguishability in chemistry.[33]

By that epoch, close contact with Argentinian philosophers and physicists, such as Federico Holik, Graciela Domenech, Christian de Ronde, Olimpia Lombardi, Juan Pablo Jorge and other members of their groups of study, was established. They made frequent visits to Florianópolis and the Brazilian group has also visited them in Buenos Aires, participating in several conferences in both countries. Until today these groups are in contact and several meetings are being organized; we can say today there is a well-characterized South-Cone Group of philosophers of physics to which surely we can add Osvaldo Frota Pessoa Jr. from the University of São Paulo, Diana Taschetto, a PhD student under Pessoa Jr., and Patricia Kauark Leite, from the Federal University of Minas Gerais. You can read about some of the activities of these people online;[34] there you can also find out about their contacts with people from the University of Cagliari and from the Vrije Universiteit of Brussells.

Holik defended a PhD thesis in which he considered the theory of quasi-sets in the foundations of quantum mechanics under the supervision of G. Domenech; one of their papers shows that quasi-cardinals of finite quasi-sets can be defined;[35] independently, Arenhart got the same result.[36] Later we developed a way to con-

[33] J. Schinaider, D. Krause, *Indiscernibilidade e identidade em química: aspectos filosóficos e formais*, "Manuscrito" 2014, Vol. 37, No. 1, pp. 113–160; N.C.A. da Costa, D. Krause, J.R.B. Arenhart, J. Schinaider, *Sobre uma fundamentação não-reflexiva da mecânica quântica*, "Scientiae Studia" 2012, Vol. 10, No. 1, pp. 71–104.

[34] URL: https://quantuminternationalnet.com/Groups-and-Members.

[35] G. Domenech, F. Holik, *A Discussion on Particle Number and Quantum Indistinguishability*, "Foundations of Physics" 2007, Vol. 37, pp. 855–878.

[36] J.R.B. Arenhart, *A Discussion on Finite Quasi-Cardinals in Quasi-Set Theory*, "Foundations of Physics" 2011, Vol. 41, pp. 1338–1354.

struct quantum mechanics via the Fock spaces formalism within the theory of quasi-sets.[37]

It is worth mentioning the recent works by Eliza Wajch, from the Siedlce University, Poland, who has practically reconstructed the theory by admitting quasi-classes and improving it in several aspects; she is a critic of the notion of quasi-cardinals as usually posed in the theory and has proposed alternatives. The work is in construction and will be published next year.[38] Eliza has also presented her works on the notion of quasi-cardinals in quasi-set theory in several places around Europe.

Presently, there are many works being developed on such issues with the addition of José Acacio de Barros, a Brazilian physicist and philosopher of physics who is based at the State University of San Francisco and has worked on physical and philosophical aspects of the fundamentality of the concept of indistinguishability in quantum theories.[39]

5. Non-Individuals

Let us consider the "standard formalism" of quantum mechanics, either the non-relativistic or the relativistic view. This is what physicists call the mathematical counterpart of the theory (or theories) even if it is not *formalized* in a logical sense. It is well known that we can associate such a formalism with plenty of *interpretations*, as Pessoa Jr.'s chapter in *History and Philosophy of Physics in the South Cone* shows.[40] Let us consider here a different one, which is not very well

[37] G. Domenech, F. Holik, D. Krause, *Q-Spaces and the Foundations of Quantum Mechanics*, "Foundations of Physics" 2008, Vol. 38, No. 11, pp. 969–994; G. Domenech, F. Holik, L. Kniznik, D. Krause, *No Labelling Quantum Mechanics of Indiscernible Particles*, "International Journal of Theoretical Physics" 2010, Vol. 49, No. 12, pp. 3085–3091; J.A. de Barros, F. Holik, D. Krause, *Distinguishing Indistinguishabilities: Differences between Classical and Quantum Regimes*, Springer, forthcoming.

[38] E. Wajch, *Troublesome Quasi-Cardinals and the Axiom of Choice*, forthcoming; D. Krause, E. Wajch, *A Reappraisal of Quasi-Set Theory and Quasi-Cardinals*, forthcoming.

[39] J.A. de Barros, F. Holik, D. Krause, *Indistinguishability and the Origins of Contextuality in Physics*, "Philosophical Transactions of The Royal Society A" 2019, Vol. 377, pp. 1–13; J.A. de Barros, F. Holik, D. Krause, *Distinguishing Indistinguishabilities*, op. cit.

[40] R.A. Martins, G. Boido, V. Rodriguez, *History and Philosophy of Physics in the South Cone*, College Publications, London 2013.

known but which brings some light to the issue: the *non-individuals* interpretation of quantum mechanics.[41] As shown in the book *Identity in Physics: A Historical, Philosophical, and Formal Analysis*,[42] the standard formalism is compatible with at least two distinct and non-equivalent accounts of quantum objects. The first see them as *individuals*, entities endowed with identity in the sense that they would obey the standard theory of identity. This is possible if one restricts the states they can be in. The second interpretation is much more interesting. It says that quantum systems, and not only "particles," lack identity in the sense that the standard theory of identity does not apply in full to them. Some words are in order to explain that.

The non-individuals interpretation establishes connections between interpreting quantum theory and the metaphysics of quantum (non-)individuality. As said in *The Non-Individuals Interpretation of Quantum Mechanics*,[43] if quantum mechanics is understood as dealing with objects of a given kind, whether particles, fields or something else, it may be asked what these objects are metaphysically. This leads to questions regarding whether they are individuals or not, and if they are, which principle of individuality determines that that is the case. The non-individuals interpretation of quantum mechanics takes the relevant entities as lacking individuality, adding a further metaphysical interpretative layer over the theory's bare entities. This is known as the *received view* of quantum non-individuality.[44]

To give a short description (the details are in the mentioned references), by an *individual* we can understand something that obeys the following three conditions: (1) it is *one* of a kind; (2) it can be differentiated from any *other* individual by some condition; and (3) it can be *re-identified* as such in different contexts, that is, as being *that individual* of previous encounters. Jonathan Lowe provides examples of non-individuals, entities that fail to meet at least one of these conditions: portions of water, for instance, fail to satisfy (3).[45] The same could be

[41] For details, see D. Krause, J.R.B. Arenhart, O. Bueno, *The Non-Individuals Interpretation of Quantum Mechanics*, op. cit.

[42] S. French, D. Krause, *Identity in Physics: A Historical, Philosophical, and Formal Analysis*, Oxford University Press, Oxford 2006.

[43] D. Krause, J.R.B. Arenhart, O. Bueno, *The Non-Individuals Interpretation of Quantum Mechanics*, op. cit.

[44] See S. French, D. Krause, *Identity in Physics*, op. cit., chapter 3, for a historical overview.

[45] E.J. Lowe, *Non-Individuals*, in: *Individuals across the Sciences*, eds. A. Guay, T. Pradeu, Oxford University Press, Oxford 2016, pp. 49–60.

said of quantum entities (and Lowe acknowledges that). One of our preferred examples goes as follows. Suppose that a helium atom is in its fundamental state. Considering spin, then its two electrons are described by a vector which is the superposition of spin up and spin down for both electrons in whatever direction you chose. The vector expresses that the states of the particular electrons are entangled and the vector cannot be factorized in particular states of the electrons; so, they are indistinguishable, and *cannot* be put apart (while in the atom). But we can ionize the atom by providing it with a certain amount of energy so that one of the electrons is realized so that the atom becomes a positive ion. Later, we can proceed inversely and capture an electron again, in a way that the ion becomes a neutral atom again. Question: are the first and the second atoms *the same* atom? Is the captured electron *the same* as that which was realized? Of course, quantum theory does not answer these questions. But, if we assume that the atom and the electrons follow the standard theory of identity, we need to assume that the original atom is either identical or different from the second one (the same for the electrons). But, fundamentally, if they are two, then by STI some difference must exist, and we know that there are none. So, which case is the case we have? Impossible to say. You could relegate this situation as a typical one in quantum mechanics, where you can have $A \vee \neg A$ (namely A that the two atoms are the same and the negation says that they are different) true even if you are unable to tell which case holds.[46] But this is not all that is being considered. If $\neg A$ holds, according to the standard theory of identity, *there exists* a property holding for one of the atoms but not for the other: which one? You cannot (or should not) leave this to metaphysics, but should provide a way to (at least logically, if not physically) provide a way to discern them. But we know ever since John Dalton, that there cannot exist any differentiation between two atoms of the same substance;[47] the situation is worst for electrons, them being particles or field excitations.

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[46] See D. Aerts, L. Beltran, *A Planck Radiation and Quantization Scheme for Human Cognition and Language*, "arXiv:2201.03306," https://doi.org/10.48550/arXiv.2201.03306, who have shown that the conjunction in quantum mechanics does not act as the conjunction in classical logic. The same can be said of the other propositional connectives, quantifiers, the notion of identity and the concept of set; see D. Krause, *Non-Reflexive Logics: Logics that Derogate the Standard Theory of Identity*, forthcoming.

[47] J. Dalton, *A New System of Chemical Philosophy*, S. Russell, London 1808.

6. Extensions

In 2022, I retired from the Department of Philosophy of UFSC, but since 2019 I am a permanent member of staff of the Graduate Course in Logic and Metaphysics of the Federal University of Rio de Janeiro. There, I started working on some aspects of metaphysics and the logic of quantum theories, and already have two graduate students dealing with the subject, in particular by pursuing the construction of a *quantum mereology*, which faces difficult problems, such as the indistinguishability of parts and quantum holism, to mention just two; these ideas were posed in the article *Is Priscilla, the Trapped Positron, an Individual? Quantum Physics, the Use of Names, and Individuation*.[48] The possibility of expanding the activities to other universities in Brasil and abroad is great and this is the plan for the future.

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[48] D. Krause, *Is Priscilla, the Trapped Positron, an Individual? Quantum Physics, the Use of Names, and Individuation*, "Arbor" 2011, Vol. 187, pp. 61–66; D. Krause, *Essay on Perspectivism in the Philosophy of Science*, "South American Journal of Logic" 2019, Vol. 5, No. 2, pp. 179–195.

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