

Gerland's *Dialectica* and Paraconsistency

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1. Introduction

The present state of paraconsistent logic attests to significant development, and its maturity permits a critical historical analysis of this advance, having in view the appreciation of its historical roots and stages of formation. The aim of our general research project consists in studying how a truly paraconsistent perspective was constituted throughout the history of logic, as well as how logical principles, rules, and systems have expressed the various contemporary concepts of paraconsistency. This article is a part of this project. It is also inspired by the approach of Józef Maria Bocheński, and in a way it follows his scientific programme. In particular, we have adopted here his thesis according to which the formal aspects of logical theory are essential, decisive, and indispensable to a good historiography of logic. The notion of logical consequence and the use of principles and rules in the obtaining of valid inferences are central in this context.¹

Analyzing the historical precedents of paraconsistent logic before the 20th century, we can identify some unanswered questions, among which are the following: was there knowledge of logical rules and principles that allowed, in some

¹ J.M. Bocheński, *A History of Formal Logic*, trans. I. Thomas, New York, NY 1970, pp. 2–23.

contexts, for inconsistency to be treated without trivialization? If such principles were known, how were these “proto-principles” stated, and in what way can they be related to the logical-paraconsistent results and rules known today? In light of these questions, we may ask ourselves if logical principles and rules according to which not everything may be deduced from a contradiction, or something may be rejected, were conceived and evoked within certain contexts and theoretical traditions.

In our research, we have studied key authors of the mediaeval period, focusing on primary sources, results and scholarly literature that are related to contradiction and the principle *ex falso sequitur quodlibet* and are pertinent to the history of paraconsistent logic. Scholastic logic is marked by a close proximity of logic, grammar, and metaphysics. There was an intense debate at this time about the validity of the principle *ex falso sequitur quodlibet* (or *ex impossibili sequitur quodlibet*, or *ex contradictione sequitur quodlibet*)² in the context of theories of implication, the doctrine of topics, and obligational disputations. The analysis of positions for and against the *ex falso* is essential to the history of the paraconsistent approach in scholastic logic and in Western thought. It is important to observe here that Bocheński, in his celebrated *Formale Logik* (1956), affirms that the *ex falso* is a mediaeval contribution to logic – “This [the Aristotelian discussion of valid syllogisms based on false premises in *An. pr.* B2, 53b7–10] is not yet the scholastic principle *ex falso sequitur quodlibet*, but only the assertion that one can

² Motivated by a historical analysis, specifically by the discussions of the *ex falso* by several thinkers during the Middle Ages, in this paper – though we recognize the distinct logical nuances – we consider the expression *ex falso sequitur quodlibet* to embrace such principles as special types of the *ex falso*. As far as we know, the first thinker to use the expression *idem esse ex contradictione* was John of Salisbury, alluding to the position of Adam of Balsham’s school in the debate; see Ioannis Saresberiensis, *Metalogicon (Metalogicus)*, in: *Patrologia Latina*, Vol. 199, ed. J.-P. Migne, Paris 1815–1875, 928C–D. Also, as far as we know, it was Chris Mortensen, in his well-known book *Inconsistent Mathematics*, Dordrecht 1995, p. 2, who used the expression *ex contradictione quodlibet* in the context of paraconsistency. In 1996, Andrés Bobenrieth used the expression *ex contradictione sequitur quodlibet*; see A. Bobenrieth Miserda, *Inconsistencias ¿Por qué no? Un estudio filosófico sobre la lógica paraconsistente*, Bogotá 1996, p. 103. Maria Luisa Dalla Chiara mentions *ex absurdo sequitur quodlibet*; see M.L. Dalla Chiara, *Logica*, Milano 1974, p. 27. In fact, the expression *ex contradictione sequitur quodlibet* had been previously used in the literature by several other logicians, such as, for instance, Else M. Barth and Erik C. Krabbe, in 1982; see E.M. Barth, E.C. Krabbe, *From Axiom to Dialogue*, Berlin 1982, p. 167.

form syllogisms in which one or both premises are false, the conclusion true"³ – an observation which has been corroborated by later scholarship.

As far as we know, the first thinkers to discuss the validity of what came to be referred to as the principle *ex falso sequitur quodlibet* were Gerland of Besançon and Peter Abelard. John of Salisbury (1159) attributes to Adam of Balsham (the head of the *parvipontani*) the thesis that "from a contradiction follows the same" (*idem esse ex contradictione*), which is generalized by the *ex falso*.⁴ From the 13th century on, there was an intense debate concerning the validity of some *consequentiae*, and one finds generations of logicians quarrelling over the legitimacy of the *ex falso*. In fact, we consider that the first author to explicitly argue against the *ex falso* under a *lato sensu* paraconsistent approach was Peter Abelard.⁵ In a forthcoming work we will present our analysis of his position.

In this paper, we analyze the role of the *Dialectica* of Gerland of Besançon in the rising of discussion about the *ex falso* in the 12th century, and we interpret his position as contrary to the acceptance of the principle. We consider Gerland one of the earliest authors to prepare the path and to properly discuss the role of the *ex falso sequitur quodlibet*, making it central in the philosophical context of the time.

As this author and his work are not well known among philosophers, we decided to provide important details in this regard. In section 2, we discuss the identity of the author of the *Dialectica*. In section 3, we outline the content of the work, with emphasis on the theory of topics and on three aspects of logical theory which are important for our discussion. In section 4, we analyze Gerland's concept of consequence, fundamental to logic and to our analysis concerning paraconsistency, presenting his semantic clauses for the veracity and the falsity of a consequence. Next, we present very basic notions about paraconsistent logic and paraconsistent theories, also introducing the key concepts of relevant logics. Finally, we conclude by interpreting Gerland's position as contrary to the accept-

³ J.M. Bocheński, *A History of Formal Logic*, op. cit., p. 98.

⁴ See L. Minio-Paluello, *Twelfth Century Logic: Texts and Studies*, Vol. 1: *Adam Balsamiensis Parvipontani Ars Disserendi (Dialectica Alexandri)*, Roma 1956; and Ioannis Saresberiensis, *Metalogicon (Metalogicus)*, op. cit., 928C–D.

⁵ See E.L. Gomes, I.M.L. D'Ottaviano, *Para além das colunas de Hércules, uma história da paraconsistência: de Heráclito a Newton da Costa* [Beyond the Columns of Hercules, a History of Paraconsistency: From Heraclitus to Newton da Costa], Campinas 2017, pp. 164–181.

ance of the *ex falso sequitur quodlibet*, including him as a defender of what is nowadays considered a paraconsistent approach in the broad sense.

In accord with Yukio Iwakuma, we assume that Gerland of Besançon, a contemporary of Abelard, wrote his *Dialectica* some years before Abelard's writings were produced. Although we are aware of the polemic concerning the authorship of some of Abelard's works, we have decided not to discuss this question here and have simply assumed that Abelard's *Editio super Porphyrium* was known during the period we are analyzing and is posterior to the publication of Gerland's *Dialectica*.⁶

In the development of our work we have used Garlandus Compotista's *Dialectica*, edited by Lambertus Marie De Rijk.⁷ In this paper, we have opted to use quotations from Eleonore Stump and Ivan Boh, well-known scholars of mediaeval philosophy and logic, who dedicated themselves directly to the study of the development of the logical-philosophical theories of the period and in particular to the study of Gerland's contributions. We quote their accurate translations in specific excerpts on the notion of consequence that we consider significant for our analysis of a possible paraconsistent approach in the *Dialectica* of Gerland of Besançon. However, as far as we know, Stump and Boh do not present approaches concerning either relevance or paraconsistency in their analyses of the *Dialectica*, and we do not know of any other authors that have explicitly analyzed Gerland's approach as being paraconsistentist *lato sensu*. It is precisely such an analysis that we consider the specific contribution of this article.

2. Gerland of Besançon

Gerland "the Computist" was for a time, due to the attribution of De Rijk,⁸ considered the author of the *Dialectica*, an important treatise of scholastic logic and

⁶ Y. Iwakuma, "Vocales," or *Early Nominalists*, "Traditio" 1992, Vol. 47, pp. 53–54. See, e.g., C.J. Martin, *A Note on the Attribution of the "Literal Glosses" in Paris, BnF, lat. 13368 to Peter Abaelard*; and M. Cameron, *Abelard's Early Glosses: Some Questions*, both in: *Arts du langage et théologie aux confins des XIe–XIIe siècles: textes, maîtres, débats*, ed. I. Rosier-Catach, Turnhout 2011, pp. 605–646 and pp. 647–662, respectively.

⁷ Garlandus Compotista, *Dialectica*, ed. L.M. De Rijk, Assen 1959.

⁸ De Rijk's hypotheses about the identity of Gerland (*Introduction. Part I: The Author of the Dialectica. His Life and Works*, in Garlandus Compotista, *Dialectica*, op. cit., p. xlv) are, in short, the following: "It has been shown in the first part of this Introduction that the master Gerland

one of the oldest extant in complete form.⁹ Recently, based on new findings and improved knowledge of authors and works of the period, Iwakuma has proposed a complete revision of both the attribution of authorship and the period to which this work in fact belongs.¹⁰

According to Iwakuma, the authorship of the *Dialectica* can be ascribed to one of two Gerlands, an older or a younger. De Rijk attributed it to the former, who is known as Gerland "the Computist." However, the Elder Gerland has now been identified as Saint Gerland, bishop of Agrigento, who died on 25 February 1100. Iwakuma argues that the Younger Gerland, now also credited with authoring the *Candela*,¹¹ is the author of the *Dialectica*.

Iwakuma's first argument is based on the fact that the earliest records of the vocalist doctrine appear only in 1080. The *Dialectica* exhibits unmistakable traces of its author being a vocalist (an early nominalist); therefore, the work could not have been written before 1075, and certainly not before 1040 as suggested by De Rijk.¹² Iwakuma further argues that if the text had been written during the last two decades of the 11th century, it does not seem likely that the Elder Gerland would have embraced, at an advanced age, such an innovative vision of a topic so well established in the *logica vetus*.¹³ On the contrary, the *Dialectica* must have been composed, according to Iwakuma, not before c. 1100 and not after c. 1130.

There are two fundamental lines of evidence for determining this time limitation. The first is that in Gerland's *Dialectica* one can find traces of the *logica nova* and, in particular, indications of some awareness of Aristotle's *Topics*. The

named in the title of the Fleurian manuscript must be the eleventh century computist Garlandus, who was *magister scholarum* at Besançon at the end of his life (c. 1080 A.D.). I propose to call him Garlandus Computista (c. 1015–before 1102)." A computist is a person who is skilled in computing, for instance, calculating the dates of the calendar using astrometry and celestial mechanics. As De Rijk (*Introduction...*, op. cit., p. xxii) explains: "Computus' was the name given in the Middle Ages to what was considered to be the most important branch of astronomical science. It was closely allied to the science of modern almanack-makers, its object being to calculate the Year of Grace and also dates of moveable feasts of the Church, especially that of Easter, by the motions of the sun and the moon. It also served scientific chronology in general."

⁹ See L.M. De Rijk, *Introduction...*, op. cit., p. xlix; and E. Stump, *Dialectic in the Eleventh and Twelfth Centuries: Garlandus Computista*, "History and Philosophy of Logic" 1980, Vol. 1, p. 2.

¹⁰ See Y. Iwakuma, "Vocales," or *Early Nominalists*, op. cit., pp. 37–111.

¹¹ See *ibid.*, p. 48. *Candela* is an encyclopedic work on dogmatic theology, liturgy, and canon law (cf. L.M. De Rijk, *Introduction...*, op. cit., p. xxxii).

¹² De Rijk, when preparing the critical edition of the manuscript, considered that the work should be dated to after c. 1015 and before 1102.

¹³ Y. Iwakuma, "Vocales," or *Early Nominalists*, op. cit., p. 48.

first texts that document this awareness are the *Logica "Ingredientibus"* of Peter Abelard (c. 1117/1121), the *Ars disserendi* of Adam of Balsham (written in 1132), as well as other texts from the early 12th century, which demonstrate that some scholars had early access to manuscripts of the *Prior Analytics*. Thus Iwakuma concludes that Gerland's *Dialectica* could not have been written before 1100.¹⁴

The second line of evidence for this dating, Iwakuma argues, lies in certain terminological parallels between Gerland's *Dialectica* and Abelard's *Logica "Ingredientibus"*. The latter work did not circulate before 1120. The use by Gerland of the term *status* in the *Dialectica* is thus coherent with what is found in the texts of the first decades of the 12th century in which this term appears. This employment of the term, however, as Iwakuma explains, is not completely technical, as in Abelard, but it does clearly suggest that Gerland's *Dialectica* predates Abelard's celebrated work. In addition, similar parallels are found between the *Dialectica* and Abelard's *Editio*.¹⁵ Iwakuma argues that "[i]n Abelard's *Editio super Porphyrium* as well as in Gerland's *Dialectica*, the *numerus* is glossed as *collectio* of accidents peculiar to an individual."¹⁶ Therefore, asserts Iwakuma, "[i]f we can conclude from these facts that Abelard had read Gerland's *Dialectica* by the time he wrote his *Editio* in ca. 1102/1108, Gerland's work should be dated no later than the first years of the twelfth century."¹⁷

If Iwakuma's proposed date for the *Dialectica* is correct, its author cannot be the Elder Gerland, the computist, because St Gerland died in 1100. The author would then be, according to Iwakuma, the Younger Gerland, who appears in documents from 1131 to 1134 as prior of the regular canons of St Paul of Besançon.¹⁸ Iwakuma believes that there is no way to definitively decide the authorship; however, he lists other reasons for concluding in favour of Gerland of Besançon.¹⁹

¹⁴ Ibid., pp. 49–50.

¹⁵ The authenticity of this work has been questioned. See our observation in sect. 1.

¹⁶ Y. Iwakuma, "Vocales," or *Early Nominalists*, op. cit., p. 53.

¹⁷ Ibid. (our emphasis).

¹⁸ Ibid., pp. 53–54.

¹⁹ The rationale for Iwakuma's attribution (ibid., p. 54) takes into account the following: (i) there is a letter from Roscelin to Abelard, dating from when the former was canon of Besançon, Tours, and Loches. At that time, explains Iwakuma, "Gerland may, then, have been a pupil of Roscelin's in Besançon," where he would have had contact with the vocalist doctrine; and (ii) in the *Gesta Alberonis archiepiscopi Balderico* it is recorded that in 1147 the Archbishop of Trèves invited Gerland together with Thierry de Chartres to join him on a trip to Frankfurt. Iwakuma concludes (ibid.): "It might be suggested that Gerland and Thierry had been friends earlier. Both men belonged to the first generation to whom the *logica nova* was accessible. And it may deserve

3. Gerland's *Dialectica*

Gerland of Besançon (died c. 1148) is an important author for understanding the development of certain typically mediaeval logical theories. His work provides an important record of a series of logical doctrines at an early stage of development. De Rijk prepared the edition of the *Dialectica* of Garlandus Compotista from the two remaining manuscripts, Paris BnF Lat. 6438 and Orleans 260(216). According to De Rijk, "Both manuscripts contain the complete text of a *Dialectica* in six books, which, though mainly based on Boethius' logical works, give a rather independent exposition of the *logica vetus*."²⁰ In this regard, explains Stump, "[d]ialectic in the eleventh to twelfth centuries, though it derives largely from Boethius's work and is couched mainly in his terms, is very different from the method and theory of dialectic in Boethius."²¹ Furthermore, Stump writes, "[a]ll of this should give the impression that Garlandus's treatment of dialectic closely resembles Boethius's, and in many respects it does. The dissimilarities, however, are many and important."²² Thus, although his treatise includes logical doctrines that are standard for the period, Gerland innovates by paying special attention to hypothetical syllogisms, as well as to the study of consequence, a logical doctrine that, as we shall see, he intertwines with the theory of topics.

Gerland's *Dialectica* reflects and portrays the creative dawn of mediaeval scholastic logic.²³ Book by book, the author develops the doctrines of his theoretical framework, placing them at the core of the *logica vetus*, but with hints of the *logica nova*. In the first book, on non-complex voices (*De vocibus incomplexis*), Gerland discusses the categorematic part of language as it relates to logic, analyzing its simplest elements, the non-compound voices (*voces*), as they relate to the

our attention that Thierry is likely to have been a friend of Abelard's as well, since he sided with him at the Council of Soissons in 1121. Is it mere coincidence that Abelard had knowledge both of the *logica nova* and of Gerland's work, as I have argued above?"

²⁰ L.M. De Rijk, *Introduction...*, op. cit., p. ix. It is worth noting that the sources of the *Dialectica* not only include Boethius's translations and commentaries on the logical treatises of Aristotle, but also works such as Boethius's *De hypotheticis syllogismis*. In the latter figure elements of Megaric-Stoic logic, which are an important starting point for the mediaeval analysis of the conditional proposition, the notion of implication, and logical consequence (see *ibid.*, pp. xlvi–xlix).

²¹ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 2.

²² *Ibid.*, p. 3.

²³ Stump (*ibid.*, p. 2) argues that "it [the *Dialectica*] stands at the beginning of the scholastic tradition, and many of the controversies and doctrines of later scholastic work on dialectic are prefigured in or derived from the philosophical tradition represented by Garlandus."

terms that can appear in a proposition, that is, the five predicables of Porphyry and the ten traditional categories of Aristotle; all of these items are understood in the light of a nominalism that is coherent with Gerland's general attitude towards the purpose and object of logic. In the second book, on complex voices (*De vocibus complexis*), Gerland presents the theory of enunciative propositions and their respective types, as well as the main logical operations that can be carried out with these propositions, such as equipollence, opposition, and conversion of categorical propositions.²⁴ In the third book, on univocal and multiple propositions (*De propositione una et multiplici*), Gerland analyzes the properties of the constitutive elements of the proposition, presenting a semantic theory of univocal propositions (whose terms are univocal) and multiple propositions (whose terms admit multiple meanings); this book also discusses the definition and classification of modal propositions.²⁵ In the fourth book, on the differences of topics (*De topicis differentiis*), Gerland presents a classification of species of arguments – syllogism, induction, enthymeme, and example – and a series of topics, grouped with an emphasis on their differences.²⁶ In the fifth book, Gerland discusses categorical syllogisms (*De sillogismis cathgoricis*), presenting them in three figures, analyzing how the valid modes of the second and third figures can be reduced to the valid modes of the first figure. However, it is in the sixth book of the *Dialectica*, dedicated to the study of hypothetical syllogisms (*De sillogismis hipoteticis*), that Gerland reveals interests that anticipate and record, from a historical point of view, certain logico-theoretical tendencies that became more pronounced, and

²⁴ Gerland does not offer a clear and categorical definition of equipollence. However, from the use of this notion by the author in passages in the *Dialectica*, equipollence corresponds to the logical equivalence relation between two propositions (in today's sense). Regarding the notion of equipollence, Terence Parsons (*Articulating Medieval Logic*, Oxford 2014, p. 60, n. 6) explains: "The term 'equipollent' is not usually defined, but it seems to just mean the logical equivalence between two propositions. So understood, simple conversion produces equipollent propositions, but I am not aware of anywhere that the term is used to characterize the results of conversions."

²⁵ Garlandus Compotista, *Dialectica*, op. cit., 76.7–14. Throughout this paper we will refer to mediaeval authors in accord with the pattern "x.y-z," where "x" refers to the page number and "y" and "z" denote the corresponding lines.

²⁶ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 7: "It is plain that for Garlandus, as for Boethius, *Differentiae* are the more important of the two sorts of Topics. *Differentiae* can be thought of, roughly, as the headings under which maximal propositions can be grouped. Some maximal propositions are generalizations about definition, so *definition* (or *from definition*) is a *Differentia*; other maximal propositions are about opposites, so *opposites* (or *from opposites*) is a *Differentia*; and so on."

no less important, in the following centuries. Among them may be highlighted the author's effort to expound the theory of the hypothetical syllogism. In this text, at a very early stage of scholastic logic, Gerland sought by means of topical analysis to establish the logical criteria for the notion of consequence. Even from the point of view of the space it takes up in the text, this discussion stands out: about a third of the *Dialectica* is dedicated to it (Book VI, 63 pages, on hypothetical syllogisms).²⁷

Three aspects of the logical theory of the *Dialectica* are important for our discussion: (i) the way the author approaches certain topical inferences with a view to selecting logically acceptable inferences; (ii) his analysis of the notion of logical consequence; and (iii) the definition of implication that may be deduced from his notion of consequence. In what follows, we will try to show whether these three points can or cannot support an interpretation according to which Gerland subtly anticipated discussions about the general invalidity of the *ex falso sequitur quodlibet* principle. This principle, perhaps unknown (or not ostensibly stated) at the time when Gerland was writing, would later be the object of lively controversy in the 12th century.

Before we proceed, it is important to take note of an important stylistic characteristic of the logical treatises of the early scholastics. In the works of both Gerland and Abelard, as in the case of other 12th-century logicians, there is a peculiar literary style, which is markedly different from that found in the exposition of logical theories in the following centuries. In fact, as Brian P. Copenhaver, Calvin G. Normore, and Terence Parsons explain,

[t]welfth-century logicians [...] were disorderly, noisy, and polemical, recording their fights in a genre of books that have been described as catechisms or manifestos. Usually they carry the name of one of the competing schools: they promote the logical principles characteristic of that school; they set problems to be solved by its principles; and sometimes they try to show why a different

²⁷ For comparative purposes, note that De Rijk's edition of the *Dialectica* totals 181 pages, distributed as follows: Book I, 39 pages; Book II, 21 pages; Book III, 19 pages; Book IV, 28 pages; Book V, 11 pages; Book VI, 63 pages. This outline illustrates the extent to which the sixth book exceeds the size of the others. As Boh explains in *Epistemic Logic in the Later Middle Ages*, London 1993, p. 6: "Observing that Garland's *Dialectica* contains thirty-eight [sic] pages on topics (*loci*), sixty-three pages on hypothetical syllogisms, and only eleven pages on categorical syllogism helps us to realize how important it was to him to find a viable definition or a description of consequence."

school cannot solve such problems. But logic in this period was belligerent not just in these catechisms but also in textbooks and lecture courses like Abelard's, which often name their rivals and attack rival doctrines.²⁸

This polemical and combative characteristic offers additional difficulties in the hermeneutic reconstruction of these authors' logical theories. Thus, in order to recompose this doctrinal mosaic, it is necessary to examine a series of textual elements that, little by little, can allow us to understand the points we wish to focus on in these works.

Firstly, Gerland's formal approach to logic draws our attention, as it anticipates a perspective that would become increasingly common from then on.²⁹ Thus, De Rijk explains, "[h]e attributed to logic a merely formal task: it only aims, in his opinion, at distinguishing valid arguments from invalid ones and to state *why* they are valid or not. The discovery of truth is of secondary importance to him."³⁰ Gerland understands logic as *ars sermocinalis*, that is, as a science aimed at analyzing the structure of language rather than defining the status of elements of logic in the science of reality or mind.³¹ Second of all, note that the division of dialectics assumed by Gerland maintains links with the previous tradition. In that sense, explains Stump, "Garlandus opens the chapter by paraphrasing Boethius's very imposing division of logic into evaluation of arguments (*iudicium*) and discovery of arguments. It is a distinction that Garlandus not only

²⁸ B.P. Copenhagen, C.G. Normore, T. Parsons, *Introduction*, in: Peter of Spain, *Summaries of Logic*, Oxford 2014, p. 13. The scholars conclude: "We find nothing like this in Peter's *Summaries*, where the author rarely seems to address an opponent, and then only quietly and obliquely – with a single marginal exception" (ibid.).

²⁹ Note E. Stump's, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 13, interesting remark concerning the style Gerland uses when analyzing whether some instances are good candidates for being sound logical inferences: "The oddness of Garlandus's examples is mystifying until we see that he is not interested in this or that particular question or conclusion but in the forms of acceptable inferences. He is not interested in settling issues about man's whiteness; his concern is with all inferences of the type 'If every animal is _____, then man is _____', where the same expression is to fill both blanks. Given Garlandus's concern, it is not unlikely that he deliberately chooses apparently inane examples."

³⁰ L.M. De Rijk, *Introduction...*, op. cit., p. lii.

³¹ In this respect, see E.A. Moody, *Truth and Consequence in Mediaeval Logic*, Amsterdam 1953, pp. 5–6; see also P. Thom, *Robert Kilwardby's Science of Logic: A Thirteenth-Century Intensional Logic*, Leiden–Boston, MA 2019, pp. 14–17. Note De Rijk's observation (*Introduction...*, op. cit., p. liii) that "[i]t is indeed a remarkable fact that we do not find any trace of the controversy between realists and nominalists in the *Dialectica* of Garland. He shows himself a consistent nominalist, which can be understood easily because of his conception of logic as an *ars sermocinalis*."

preserves but also takes seriously; and, like Boethius, he relegates dialectic to discovery."³²

In absorbing the theory of topics from the previous tradition, especially that of Boethius's commentaries on Aristotle's *Topics* and on Cicero's *Topics*,³³ scholastic logic introduces perceptible changes. Contrary to the ancient tradition, in which topics were seen as instruments for the discovery of arguments, topics in scholastic logic are thought of as instruments for the justification of inferences.³⁴

Gerland closely follows the systematization of topics in the manner of Boethius, but with notes of originality. For Gerland, topics, understood as *maxima propositio*,³⁵ are instruments for assessing the validity of arguments.³⁶ This is an important theoretical feature in the systematization of the logical theories of the period. An epistemic tone dominates in Gerland's definition of argument. On

³² E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 3. It was not by chance that, during the development of scholastic logic, the chapters related to dialectic in logical treatises were absorbed into a general theory of consequence. Stump (ibid., p. 2) explains that "[u]nder its impetus the study of dialectic developed and changed until dialectic became absorbed into the theories of consequences or conditional inferences important in the fourteenth and fifteenth centuries."

³³ Boethius's "In Ciceronis Topica", trans., notes, and essays on the text E. Stump, Ithaca, NJ–London 2004.

³⁴ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 1, explains that "[b]oth Aristotle and Boethius think of Topics as instruments for a logic of the discovery of arguments. The scholastic use and understanding of Topics is very different from that of Aristotle or Boethius, though the scholastic tradition of dialectic is by no means uniform."

³⁵ As argued in L. Gili, P. Podolak, *Hugh Eterianus, Alexander of Aphrodisias and Syllogistic Demonstrations: A Newly Discovered Fragment of Alexander of Aphrodisias' "Commentary on Aristotle's Posterior Analytics"*, "Documenti e Studi sulla Tradizione Filosofica Medievale" 2018, Vol. 29, p. 151, Boethius had already translated ἀξιωμα as *maxima propositio*: "In his translation of Aristotle's *Topics*, Boethius had already translated ἀξιωμα in this way (cf. Arist., *Top.* VIII, 1, 155 b15, in: *Topica*, transl. A.M.T.S. Boethius, ed. L. Minio-Paluello, Desclée de Brouwer, Paris 1969, p. 156). [...] Accordingly, we maintain that the best English translation of *maxima propositio* is simply 'axiom' in this context."

³⁶ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 3, explains Gerland's view: "One sort of Topic, he says, is a maximal proposition, which he defines as Boethius does: a maximal proposition is a self-evidently true proposition, for which no proof can be found and which can serve as the basis of proof for other propositions ([*Dialectica*] 87.4–8). Although he says Topics belong to the part of logic concerned with discovery ([*Dialectica*] 86.14), it is important to notice that he assigns maximal propositions to the part of logic concerned with evaluation ([*Dialectica*] 86.12)"; for instance, as Stump (ibid., pp. 3–4) explains, when Garlandus affirms that "a maximal proposition proves a syllogism ([*Dialectica*] 86.12–13)." In this case, Stump (ibid., p. 14) indicates, "Categorical syllogisms [...], he says, 'are aided by the Topics from the whole and from the part and from an equal' ([*Dialectica*] 114.18)?"

this point, he follows Boethius closely, because, as Stump explains, “[i]n his discussion of the nature of an argument, Garlandus (following Boethius) defines argument as a reason producing belief concerning something that was in doubt ([*Dialectica*] 92.19).”³⁷ For Gerland, the role of an argument is to produce a correct opinion about something that is in dispute or needs to be known. One of his great innovations is the importance placed on conditional propositions throughout the logical theory of the *Dialectica*.³⁸

Thus, although the general theory of topics belongs, according to Gerland, to the discovery of the conclusions of the arguments, this is, claims Stump, a subtle change introduced by the mediaeval authors in the topical tradition inherited from the ancients, so that other logical theories, such as the theory of the syllogism (categorical and hypothetical), are subordinated to them; furthermore, of the two types of topics, Gerland, like Boethius, chooses the *differentiae* as the most important item in topical theory. The *differentiae* are, in a general and simplified manner, titles under which maximal propositions are grouped. In what follows, Gerland exhibits the topical foundation of *modus ponendo ponens* and *modus tollendo tollens* and their maximal propositions:

No one should be surprised if one topic is called by different names, just as the topic “from the adjacencies,” which is also called “from common accidentals” by different [authors], while all topics can be named either “from the antecedent” or “[from the] consequent.” From the antecedent it is as follows:

“If she gave birth, she lay with a man;
but she gave birth,
therefore, she lay with a man,”

the topic from the antecedent: maximal proposition [axiom], once the antecedent is established, the consequent is established.

³⁷ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 4. The original runs as follows: “Argumentum est ratio rei dubie faciens fidem” (*Dialectica* 92.19).

³⁸ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 12: “What Garlandus himself is really interested in is the Topics’ usefulness for the analysis of conditional propositions. The Differentia finds or provides a conditional premise, and that conditional premise is provided by the maximal proposition in a categorical argument which has the conditional as conclusion.”

From the consequent it is thus:

"but she didn't lie with a man,
therefore, she didn't give birth,"

the topic from the consequent: maximal proposition [axiom], once the consequent is annihilated, the antecedent is annihilated.

You should know that all the topics are in the service of hypothetical syllogisms; but only the topics "from the whole," "from the part" and "from an equal" are of service to categorical [syllogisms].³⁹

Moreover, Stump suggests: "It seems to me just possible that Garlandus is thinking of this broad function of these ultimate maximal propositions when he says that maximal propositions contain or constitute the sense of an argument."⁴⁰ Thus Gerland attributes to the topic from the antecedent and to the topic from the consequent, respectively equivalent to the rules of *modus ponens* and *modus tollens*,⁴¹ the important role of grounding *all* hypothetical inferences.⁴²

³⁹ *Dialectica* 114.3–18, our translation. The original runs as follows: "Nemini mirum videatur, si idem locus diversis nominibus appellatur, sicuti locus ab adiunctis appellatur etiam a communiter accidentibus secundum diversos, cum omnes loci appellari possint ab antecedenti vel consequenti. Ab antecedenti sic: 'si peperit, cum viro concubuit; atqui peperit igitur cum viro concubuit', locus ab antecedenti; maxima propositio: posito antecedenti ponitur consequens. A consequenti sic: 'sed non concubuit, non igitur peperit', maxima propositio: destructo consequenti destruitur antecedens. Sciendum est quod omnes loci serviunt hipoteticis sillogismis; catheticis vero tantum serviunt locus a toto et a parte et a pari." The propositions employed by Gerland in his examples of *modus ponens* and *modus tollens* come from Cicero's *De inventione* (I.70–72); see Cicero, *On Invention, The Best Kind of Orator, Topics*, trans. H.M. Hubbell, Loeb Classical Library 386, Cambridge, MA 1949, pp. 118–119.

⁴⁰ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 14, n. 31.

⁴¹ It should be remembered that both *modus ponens* and *modus tollens* belong to the Stoic tradition, which inaugurated logical-propositional analysis. There they appear as the first and second indemonstrables, respectively. Such results reached the mediaeval authors, as far as we know, mainly through the works of Boethius.

⁴² E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 14, explains that Garlandus deepened the link between the theory of the topics and of conditional propositions: "Garlandus goes even further along this line by claiming that all Topics can be subsumed under two most general Topics, the Topic *from the antecedent* and the Topic *from the consequent*. Corresponding to each of these is a maximal proposition; as Garlandus gives these, they are equivalent to the rule for *modus ponendo ponens* and the rule for *modus tollendo tollens*, respectively ([*Dialectica*] 114.3–16). These are, of course, basic axioms for all hypothetical arguments; and the fact that Garlandus lists them as Topics strongly suggests that he thinks all hypothetical argumentation is dependent for its validity on the Topics."

4. Gerland's Concept of Consequence

The notion of consequence is fundamental to logic. The search for a general definition of consequence was as important to Gerland as it was to Abelard and other authors of the 12th century. In fact, as Boh explains, “[a]lthough the idea of consequence must be as old as the idea of logic itself, Garland pays more attention to it than the logic textbooks of the *summulists* of the thirteenth century.”⁴³ We can therefore reconstruct the author’s perspective from other items of his exposition because, as the scholar explains, “Garland does not offer a definition of consequence as such, but he does discuss truth and falsity conditions of it in some detail, and he is helpful with providing examples.”⁴⁴ However, one must keep in mind, as Stump argues, that for Gerland “[t]he complete argument would consist of this conditional as a first premise, the assertion of the conditional’s antecedent as a second premise, and the consequent of the conditional as the conclusion.”⁴⁵ Such a structure fits perfectly with the configuration of the rules of *modus ponens* and *modus tollens* – rules of inference that are undeniably Stoic, as previously noted.

As hypothetical inference and its vehicle – the conditional proposition – are central for Gerland, the conditions of truth and falsehood become crucial in order for us to get to the heart of his notion of consequence. He states in the *Dialectica* that a consequence is true in four ways:

⁴³ I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 4.

⁴⁴ *Ibid.*

⁴⁵ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 9. Stump details Gerland’s method: “In practice, what Garlandus draws is a conditional proposition. But since the argument he has in mind is a simple hypothetical syllogism, all he really needs to find is a suitable conditional proposition. The question determines the conclusion (or, more precisely, the categorical terms of the conclusion) since the conclusion must constitute a positive or a negative answer to the question. And given the conditional proposition and the conclusion, the second premise of the argument will be obvious” (*ibid.*). This strategy is inherently linked to logical-topical methods because, as Stump (*ibid.*, p. 16) explains, “[f]or Boethius, the main function of the Topics is discovery, and what they aid in discovering are third terms around which categorical arguments can be built. For Garlandus, the important function of the Topics is confirmation; and although they confirm all inferences on his view, he is especially interested in them in so far as they confirm enthymematic inferences in conditional propositions and so help determine the truth or falsity of premises in hypothetical syllogisms.”

- [GAR-T1] One is composed of two true propositions, as in "If Socrates is a man, he is an animal"; for both of these are true, "Socrates is a man" and "Socrates is an animal."
- [GAR-T2] Another is composed of two false propositions, as in "If Socrates is a stone, he is inanimate"; for both (components) are false.
- [GAR-T3] Another one is composed of a false antecedent and a true consequent, as in "If Socrates is an ox, he is an animal"; for it is false to say that Socrates is an ox and true that he is an animal.
- [GAR-T4] Still another is composed of parts neither of which is true or false, such as you can discern in this example: "If it were a man, it would be an animal"; neither of these is true or false.⁴⁶

A close look at these semantic clauses reveals more than can be concluded at first glance, as, for example, claiming that they only govern a classic notion of consequence or logical implication.⁴⁷ The first three clauses exemplify a pattern of logical relationship between general terms in which the minor is assimilated into the major and the inferior into the superior, and in which the species is included in the genus and the part in the whole. As Boh explains, a consequence such as "If Socrates is a man, Socrates is an animal" can be demonstrated to be necessary by assuming a first-order premise of the type "Every man is an animal," or, as we

⁴⁶ Garlandus Compositista, *Dialectica*, op. cit., 136.28–137.1. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 5. The original runs as follows: "Consequentia quattuor modis sit vera, alia ex utrisque veris, ut ista: 'si Socrates est homo, est animal' – vere enim utraque sunt 'Socrates est homo', 'Socrates est animal' – alie ex utrisque falsis, ut hec: 'si Socrates est lapis, est inanimatum' – utraque sunt false –, alia ex falso antecedenti et vero consequenti, ut hic: 'si Socrates est bos, et est animal' – falsum est enim dicere Socratem esse bovem et verum est esse animal –, alia ex utrisque terminis neque veris neque falsis, quem admodum in ista potest dinosci: 'si esset homo, esset animal': neuter namque veris est neque falsus."

⁴⁷ Like other mediaeval authors, Gerland considers, without distinction, conditional propositions to be arguments and arguments to be conditionals. In this sense, explains Stump (*Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 11, n. 19), "[i]t is worth remembering in this connection, though, that in various places Garlandus refers to the conditional premisses alone as arguments; see, for example, [*Dialectica*] 102.20–28, 105.31–32, 106.4–7 and 106.29–31." However, other logicians of the 12th century, Peter Abelard for instance, made a clear distinction between these notions; see C.J. Martin, *William's Machine*, "The Journal of Philosophy" 1986, Vol. 83, No. 10, p. 569.

have already noted, using a result of a metalogical character, such as the topical rule “Of whatever a species is predicated, of its genus is also predicated.”⁴⁸ In this case, Boh suggests that, “[c]onsidering that ‘Socrates’ is simply an instance of general terms, we can think here more of a generalized conditional structure of the form: Given that ‘man’ involves ‘animal’, it follows that ‘If x is a man, then x is an animal.’”⁴⁹ This interpretation may be added to Stump’s conclusion⁵⁰ that Gerland employs topical rules, the logical properties of conditional and categorical propositions, and the semantic clauses of his notion of consequence, for justifying results in his logical-dialectic theory. Such a practice was not so uncommon, and it demonstrates how mediaeval authors used this theoretical tool to develop their expositions of, and innovations in, logical theory.

We propose, in Table 1, a formalization of Gerland’s four statements in a first-order language, where “T” denotes “true”; “F” denotes “false”; “I” denotes “indeterminate”; “ \subset ” denotes “is strictly included in” (or “is a proper subset of”); “ \diamond ” denotes “it is possible that”; “Ms” denotes “Socrates is a man”; “As” denotes “Socrates is an animal”; “Ss” denotes “Socrates is a stone”; “Ins” denotes “Socrates is inanimate”; “Os” denotes “Socrates is an ox”; “ $\diamond Mx$ ” denotes “it is possible that x is a man”; “ $\diamond Ax$ ” denotes “it is possible that x is an animal”; “M” denotes the “set of all men”; “A” denotes the “set of all animals”; “S” denotes the “set of all stones”; “In” denotes the “set of all inanimates”; “O” denotes the “set of all oxen”; “ $M\diamond$ ” denotes the “set whose elements are possibly humans”; and “ $A\diamond$ ” denotes

⁴⁸ In this regard, Boh (*Epistemic Logic in the Later Middle Ages*, op. cit., p. 10) states: “Otto Bird, in his attempt to scrutinize the whole mechanism of the topics or *loci*, considers the relation of the species to genus as one of class inclusion, and the above maxim as a thesis of the logic of classes:

$$(A \subset B) \rightarrow (\forall x) (x \in A \rightarrow x \in B)$$

The consequence ‘If it is a man, it is an animal’ is simply an instance of it and the *locus* can now be seen as a seat of arguments and the power of inference (*vis inferentiae*.)” Boh points out that Stump, the leading investigator of the topical tradition, approves this understanding despite some reservations.

⁴⁹ *Ibid.*, p. 4.

⁵⁰ Stump affirms (*Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 9): “Garlandus in several places refers to the conditional propositions, given as examples for the *Differentiae*, as arguments (for example, ‘If the whole house is white, the wall is also white’). But he also indicates in various places that he thinks of the questions to be settled by Topical arguments as categorical questions, that is, questions in which a categorical proposition is in doubt. These two considerations strongly suggest that the arguments that Garlandus is thinking of, when he says that arguments are drawn from the *Differentiae*, are simple hypothetical arguments consisting of one conditional premise and one categorical premise; these premises together prove the categorical conclusion by *modus ponendo ponens* or *modus tollendo tollens*.”

the "set whose elements are possibly animals." Note that " $\langle X, Y \rangle \Rightarrow Z$ " must be understood as "If the truth value of the antecedent is X and the truth value of the consequent is Y , then the truth value of the conditional is Z ."

Table 1. Modes of true consequence in Gerland's *Dialectica*

Clause	Example/statement	Valuation	Formalization	Basis
GAR-T1	If Socrates is a man, he is an animal	$\langle T, T \rangle \Rightarrow T$	$Ms \rightarrow As$	$M \subset A$
GAR-T2	If Socrates is a stone, he is inanimate	$\langle F, F \rangle \Rightarrow T$	$Ss \rightarrow Ins$	$S \subset IN$
GAR-T3	If Socrates is an ox, he is an animal	$\langle F, T \rangle \Rightarrow T$	$Os \rightarrow As$	$O \subset A$
GAR-T4	If it were a man, it would be an animal	$\langle I, I \rangle \Rightarrow T$	$\diamond Mx \rightarrow \diamond Ax$	$M_{\diamond} \subset A_{\diamond}$

We claim that the fundamental characteristic of the notion of consequence in Gerland's *Dialectica* is its connexive feature, that is, his notion of logical consequence requires that the consequent of a true declarative conditional sentence must be related to the antecedent, that is, it requires a containment relation between the antecedent and the consequent. This conclusion is solidly based on other passages of the *Dialectica*. It is anchored, for now, in the clause GAR-T4, according to which, even in the case when neither the antecedent nor the consequent is true or false – we are considering that both are indeterminate – the conditional is true.

Decisive elements for the intrinsic connection of the consequence relation in Gerland come to the fore when he introduces the semantic clauses in which the consequence is false:

[GAR-F1] One is false with both components being true, as in "If Socrates is an animal, he is a man."

[GAR-F2] Another consists of two false components, e.g. “If Socrates is inanimate, he is a stone.”

[GAR-F3] Still another one is made of false antecedent and true consequent, as in “If Socrates is a stone, he is a man.”

[GAR-F4] Another one is composed of two parts, neither of which is either true or false, e.g. “If Socrates were an animal, he would be a man.”

[GAR-F5] And still another one is false which has a true antecedent and a false consequent, as in “If Socrates is a man, he is a stone.”⁵¹

Now let us examine Table 2, where “ $\not\subseteq$ ” denotes “is not included in” or “is not a subset of.”

Table 2. Modes of false consequence in Gerland’s *Dialectica*

Clause	Example/statement	Valuation	Formalization	Basis
GAR-F1	If Socrates is an animal, he is a man	$\langle T, T \rangle \Rightarrow F$	$As \rightarrow Ms$	$A \not\subseteq M$
GAR-F2	If Socrates is inanimate, he is a stone	$\langle F, F \rangle \Rightarrow F$	$Ins \rightarrow Ss$	$Ins \not\subseteq S$
GAR-F3	If Socrates is a stone, he is a man	$\langle F, T \rangle \Rightarrow F$	$Ss \rightarrow Ms$	$S \not\subseteq M$
GAR-F4	If Socrates were an animal, he would be a man	$\langle I, I \rangle \Rightarrow F$	$\Diamond As \rightarrow \Diamond Ms$	$A_{\diamond} \not\subseteq M_{\diamond}$
GAR-F5	If Socrates is a man, he is a stone	$\langle T, F \rangle \Rightarrow F$	$Ms \rightarrow Ss$	$M \not\subseteq S$

⁵¹ Garlandus Compositista, *Dialectica*, op. cit., VI, 137.4–10. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 4, modified. The original runs as follows: “Quinque modis fit false consequentia: alia ex utrisque veris falsa est, ut ista: ‘si Socrates est animal, et est homo’, alia ex utrisque falsis, ut hec: ‘si Socrates est inanimatus, est lapis’, alia ex falso antecedenti and vero consequenti, ut hic: ‘si Socrates est lapis, est homo’, alia ex utrisque neque veris neque falsis, ut hic: ‘si Socrates esset animal, esset homo’, alia iterum fit falsa ex vero antecedenti et falso consequenti, ut hic patet: ‘si Socrates est homo, est lapis.’”

Altogether, the semantic clauses for the veracity and falsity of a consequence or logical implication clearly indicate that Gerland is not dealing with a classical or material notion. When the semantic clauses above are compared, they seem to suggest an incoherent scenario in terms of the definition of consequence. On the one hand, the clauses GAR-T1-T3 and GAR-F5 seem to coincide with the truth conditions of logical-classical material implication. On the other hand, GAR-T1-T4 contrast with and antagonize clauses GAR-F1-F4, which, when considered in detail, demonstrate that it is not the designation of truth values for the antecedent and consequent in conditional propositions that decides the truth of a consequence. Observe that excepting the case in which the antecedent is true and the consequent is false – when the conditional is false⁵² – in all the other cases in which the antecedent is true or false and the consequent is true or false – and in the cases when the antecedent and the consequent are “indeterminate” – the conditional may be true or false. That is, the only case in which the truthfulness of the conditional does not depend on the content of the antecedent and of the consequent is case GAR-F5, in which the antecedent is true and the consequent is false.

For Gerland, what determines the “soundness” of the consequence is whether the content of the consequent is properly contained or comprehended (understood), extensionally or intensionally, in the antecedent, as we have tried to denote in the last column of Tables 1 and 2.⁵³

Further evidence that Gerland's notion of consequence or logical implication is connexive occurs, according to Boh, in the context of the discussion of Boethius's *De hypotheticis syllogismis*, where Gerland analyzes other types of consequences when arguing:

⁵² Garlandus Compotista, *Dialectica*, op. cit., VI, 137.12–13: “Nulla consequentia facta ex vero antecedenti et falso consequenti potest esse vera.”

⁵³ In this regard, Stump (*Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 8, n. 13) argues: “Technically speaking, the Topically-dependent arguments that Garlandus gives are not enthymemes but conditional propositions. By a true conditional, however, Garlandus means more than just a conditional whose consequent is not false if its antecedent is true; he also requires as a criterion for a true conditional that the consequent can be acceptably inferred from the antecedent. The difference between enthymemes and conditional propositions is not a great one, given this view of conditional propositions.”

A connected (*connexa*) hypothetical proposition in which the conjunction “cum” (= when, if) is placed before the antecedent, sometimes has the same force as the one in which “if” (*si*) is placed before the antecedent; for example, when I say “If a man exists, an animal exists”; for each of these says: “Because (*quia*) a man exists, an animal exists.” However, sometimes it gets another force, as here, “While (*cum*) the fire is hot, the heaven is round.” For here I do not say that because the fire is hot, the heaven is round; rather, I say that heaven is round at the same time as the fire is hot.⁵⁴

In this excerpt, Gerland introduces a division of the consequence that we find in several other authors, starting with Abelard. Gerland distinguishes (1) the accidental consequence (*per accidens*), that is, one of a temporal nature, delimited at a certain time interval, from (2) the consequence according to nature, that is, a natural one. The latter is subdivided into (i) a natural consequence due to the position of the terms (*per positionem terminorum*), as when a genus follows from the species or a cause produces an effect (for example, “When [*cum*] the sun is above the earth, it is day”), and (ii) a natural consequence by the non-positioning of the terms (*per non-positionem terminorum*), as when a species follows from a genus and an effect from a cause (for example, “If [*si*] it is day, then the sun is above the earth”) whose reciprocal (“If the sun is above the earth, then it is day”) may, in some cases, be true.⁵⁵ According to Boh, *cum* denotes a type of temporal consequence and is weaker, from an intensional point of view, than those constructed with *si* and *quia*. In fact, we could compare it to the consequence that was called *ut nunc* (as of now) in later authors.

Deepening the analysis of the connected proposition and the ways in which it can be reduced to other propositions, Gerland states the equipollence between a connected proposition and a disjunctive proposition in the following manner:

⁵⁴ Garlandus Compotista, *Dialectica*, op. cit., VI, 141.7–13. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 5. The original runs as follows: “Propositio connexa in qua ponitur ‘cum’, aliquando eandem vim obtinet cum ea in qua preponitur ‘si’, ut cum dico: ‘si homo est, animal est’. Aliquando autem aliam vim optinet, ut hic: ‘cum ignis calidus est, celum rotundum est’: non enim hic dico *quia* ignis calidus est, celum rotundum esse, sed dico celum esse rotundum *eo tempore quo ignis calidus est*.”

⁵⁵ As in the examples mentioned above; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 6.

Simple connected propositions are reduced to disjunctions equipollent to them, the disjunction being formed from negation of the antecedent while the consequent remains as it is.⁵⁶

It is surprising that Gerland formulates, at an early stage of scholastic logic, the reduction of implication in terms of the disjunction of the negation of the antecedent and of the consequent, which would be perfectly valid, in truth-functional terms, in what is nowadays called classical propositional logic (in symbols, $P \rightarrow Q \Leftrightarrow \sim P \vee Q$) and is valid if we consider only his clauses GAR-T1–T3 and GAR-F5. However, even if we consider his disjunction as an exclusive disjunction, such equipollence is not in accordance with Gerland's notion of true consequence, in the sense that the content of the consequent must be properly contained or comprehended in the antecedent, as presented in clauses GAR-T1–T4 and GAR-F1–F5.

Thus, in conformity with the logical methods described earlier, Gerland recognizes as valid the equipollence of the following propositions: "If it is a man, it is an animal," "Either it is not a man or else it is an animal," and "Every man is an animal,"⁵⁷ which provides Boh with a basis for concluding that "[r]ecognition of this last kind of equipollence tends to support the interpretation of Gerland's conditionals or consequences as stronger than merely truth-functional."⁵⁸ Based on Gerland's analysis, Boh claims that "there is even a reason to think that consequence is for him a proposition which, if true, is necessarily true, and if false, necessarily false. If so, he may be at the very beginning of a prominent line of logicians who define consequence by way of an alethic modal notion of possibility, ' $P \rightarrow Q =df. \sim \diamond(P \& \sim Q)$.'"⁵⁹ Boh concisely presents the problem:

⁵⁶ Garlandus Compotista, *Dialectica*, op. cit., VI, 131.2–4. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 6. The original runs as follows: "Simplices propositiones connexe in disiunctas sibi equipollentes resolvuntur prima parte coniuncte propositionis destructa, ultima vero manente integra."

⁵⁷ Garlandus Compotista, *Dialectica*, op. cit., VI, 131.32–132.2–3: "Equipollent etiam quedam categorice quibusdam hipoteticis propositionibus tam connexis quam disiunctis. Ut hic patet: '*si est homo, est animal*', '*aut non est homo, aut est animal*', '*omnis homo est animal*'; he enim eandem veritatem tenent et per unam alie probantur. Ideoque notandum est quod coniuncte atque disiuncte simplices per universales categoricas sibi equipollentes probantur: '*si est homo, est animal*', '*aut non est homo, aut est animal*' verificantur per istam in veritate eis consimilem: '*omnis homo est animal*.'"

⁵⁸ I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 7.

⁵⁹ *Ibid.*, p. 6.

Thus, if we are to envisage any equivalence or equipollence between connective and disjunctive propositions, then we should make sure that if one side is understood modally, the other side must also be so understood. One should not forget that if $P \rightarrow Q$ is to be taken connectively as $\sim\Diamond(P \ \& \ \sim Q)$, it could not be equivalent to a truth functional disjunction of the denial of the antecedent with the consequent, i.e. to $\sim P \vee Q$. A modal disjunction is required.⁶⁰

This is perfectly possible, Boh concludes, because both Gerland and Abelard “are still under the sway of ancient intensional readings of various ‘hypothetical’ propositions [so as] to be able to meet this requirement.”

It is important to emphasize here that we do not agree with Boh’s last claim. His interpretation of Gerland’s conditional by using modal disjunction, and in terms of strict implication, does not express the containment relation that must hold between the content of the antecedent and of the consequent. In our final remarks, we will argue for a proto-relevantist-paraconsistentist interpretation of Gerland’s consequence relation, which is explicitly stated by Peter Abelard – as previously mentioned, the latter will be discussed in a forthcoming paper.

5. Paraconsistent Logic, Relevance, and Paraconsistency

Now we introduce some basic definitions in order to outline the contemporary paraconsistent logical approach.

A theory whose underlying language has a symbol for negation is *inconsistent* if there is a formula of its language such that the formula and its negation are both theorems of the theory; otherwise, the theory is called *consistent*. A theory is *trivial* if all formulas of its language are theorems.

A logical system is *paraconsistent* if it can be the underlying logic for inconsistent but non-trivial theories, which are called *paraconsistent theories*.

Taking into account the practice of Newton C.A. da Costa, Otávio Bueno and Décio Krause, our use of the terms “consistency” and “inconsistency” is syntactical, according to the original metamathematical terminology of David Hilbert and his school.⁶¹

⁶⁰ Ibid., pp. 6–7.

⁶¹ N.C.A. da Costa, D. Krause, O. Bueno, *Paraconsistent Logics and Paraconsistency*, in: *Philosophy of Logic*, ed. D. Jacquette, Amsterdam 2006, pp. 821–822.

If the underlying logic of a theory is classical logic, or another standard logic such as intuitionistic logic, inconsistency entails triviality, and conversely.⁶²

In paraconsistent logics, the scope of the *principle of (non-)contradiction* is in a certain sense restricted; and, in every paraconsistent logic, from a formula and its negation it is not possible, in general, to deduce any formula of its language. Because of this, in such logics the notions of inconsistency and triviality are, in fact, independent notions.

Thus, the *principle ex falso sequitur quodlibet* – “from falsehood, anything follows” (or *ex impossibili sequitur quodlibet* – “from the impossible, anything follows,” or *ex contradictione sequitur quodlibet* – “from contradiction, anything follows,” or the *principle of Pseudo-Scotus*, nowadays also known as the *principle of explosion*),⁶³ the law according to which any statement can be proven from a contradiction, is not valid in general in paraconsistent logics.⁶⁴

Relevance logic or *relevant logic* is a kind of non-classical logic developed as an attempt to avoid the paradoxes of material and strict implications. Relevance logic aims to capture aspects of implication that are ignored by the material implication operator in classical truth-functional logic, specifically the fact that the antecedent seems irrelevant to the consequent, that is, the fact that the conclusion seems to have nothing to do with the premise. Relevance logicians claim that it is necessary to capture the notion of relevance between the antecedent and consequent of a true implication.

The motivations and intuitions underlying relevant logic and paraconsistent logic are distinct, but an interesting property of relevant logic is that the *ex falso* is not valid in general. Hence, relevant logics are paraconsistent logics.⁶⁵

⁶² See I.M.L. D'Ottaviano, *On the Development of Paraconsistent Logic and da Costa's Work*, “The Journal of Non-Classical Logic” 1990, Vol. 7, No. 1–2, pp. 89–152.

⁶³ Cf. sect. 1, n. 2, above, for further information.

⁶⁴ See E.L. Gomes, I.M.L. D'Ottaviano, *Para além das colunas de Hércules...*, op. cit.

⁶⁵ See A.R. Anderson, N.D. Belnap Jr, *Entailment: The Logic of Relevance and Necessity*, Vol. 1, Princeton, NJ 1975; R. Routley et al., *Relevant Logics and their Rivals*, Atascadero, CA 1982; J.M. Dunn, G. Restall, *Relevance Logic*, in: *Handbook of Philosophical Logic*, Vol. 6, 2nd ed., eds. F. Guenther, D. Gabbay, Dordrecht 2002, pp. 1–128.

6. Final Remarks

The notion of logical consequence or implication assumed by Gerland is not truth-functional and requires that the premise must be relevant to the conclusion. Gerland's notion of consequence has a connexive feature, that is, his notion demands a containment relation between the antecedent and the consequent.

Our claim is that Gerland's conception of consequence has attributes of a relevance-logical nature, as it relies on the connection between the content of the antecedent and the consequent, a reliance which could impel him to not assume as valid a type of consequence like the *ex falso*, whether in a "categorical" version (*ex falso sequitur quodlibet*), or in a modal version (*ex impossibili sequitur quodlibet*), or in its particular version (*ex contradictione sequitur quodlibet*). Not even the known corollary of the *ex falso*, "the necessary follows from anything" (*necessarium sequitur ad quodlibet*), that is, a necessary proposition follows from any proposition, is found in his exposition; another variant of this statement, presented by Bocheński in his famous history of logic, "the true follows from anything" (*verum sequitur ad quodlibet*), is also not found in Gerland.⁶⁶ In fact, there do not seem to appear such typical statements, or even variations of these, in the *Dialectica*, which is in accord with the fact that the role of the intensional aspect in Gerland's logical analysis goes beyond his professed nominalism (vocalism).

Gerland does not admit, as far as we know, that from the false or from a contradiction there follows any other proposition whatever, as the author himself repeatedly rejects this throughout his exposition, stating that certain inconsistencies produce inconvenience between terms and harbour sophistry.⁶⁷ In these cases, in the numerous instances where a way out could be an appeal to some form of the *ex falso*, the author reaffirms the inconvenience of the absurdities that are being deduced and decides the logical dilemmas on the basis of a semantic analysis of the terms, on the logico-grammatical constructions involved, and, of course, on the deductive theory that he has embraced.

Gerland seems to suggest that a stricter notion of consequence is necessary. It is in the context of topical inferences and maximal propositions that we may consider that he did not accept the *ex falso*. Relevance-paraconsistent elements seem to orient Gerland's logical options.

⁶⁶ J.M. Bocheński, *A History of Formal Logic*, op. cit., p. 342.

⁶⁷ Garlandus Compotista, *Dialectica*, op. cit., 146.13–147.26, for example.

A decisive step towards including Gerland of Besançon in the illustrious line of defenders of a notion of consequence that is, by definition, paraconsistent in a broad sense, is presented in clauses GAR-F1, GAR-F2, GAR-F3 and GAR-F4 above. In fact, we have observed that, according to the sets of clauses GAR-T1–T4 and GAR-F1–F5 taken together, excepting the case in which the antecedent is true and the consequent is false – when the conditional is false – in all the other cases in which the antecedent is true or false and the consequent is true or false – and in the cases when the antecedent and the consequent are indeterminate – the conditional may be true or false.

Hence, according to these clauses, there are circumstances in which a false antecedent does not allow any consequence to be inferred, be it true or false, or in which from a set of false (or contradictory) premises there cannot be concluded – validly – any conclusion whatever, whether true or false. It is a tacit presentation of the statement that it is not the case that anything can be concluded from the false and, like a diamond that needs to be cut, it shines forth an *ex falso non sequitur quodlibet* as an implicit corollary to its notion of consequence.

Last but not least, the theoretical tendency represented by Gerland is important for the later development and discussion of these logical theories because, as Stump concludes,

[t]hese points of contact between the twelfth-century works and Garlandus's treatise are, of course, not nearly enough to conclude that Garlandus was a direct or indirect source for any of the twelfth-century work – he may or may not have been – but they do show at least that the tradition represented by Garlandus was influential for the authors of these works and so for the study of dialectic in the early scholastic period.⁶⁸

The theoretical tradition represented by Gerland of Besançon is in opposition to the statement of the *ex falso sequitur quodlibet*, thus marking out a paraconsistent approach *lato sensu* throughout the later Middle Ages.

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⁶⁸ E. Stump, *Dialectic in the Eleventh and Twelfth Centuries...*, op. cit., p. 18.

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Summary

An analysis of positions for and against the principle of *ex falso sequitur quodlibet* is essential to the history of a paraconsistent approach in scholastic logic and in Western thought. In this paper we analyze the role that the *Dialectica* of

Gerland of Besançon played in initiating the discussion about the *ex falso* in the 12th century, and we interpret his position as contrary to the acceptance of the principle. We consider Gerland one of the earliest authors to prepare the path and examine properly the role of the *ex falso sequitur quodlibet* principle, making it central in the philosophical context of the time. We adopt the thesis of Józef Maria Bocheński, according to which the formal aspects of logical theory are essential, decisive, and indispensable to a good historiography of logic.

Key words: theory of topics, logical implication, consequence, *logica vetus*, early scholastic logic, Gerland of Besançon, paraconsistency